TIME-TO-PRODUCE, INVENTORY, AND ASSET PRICES

A Dissertation

by

ZHANHUI CHEN

Submitted to the Office of Graduate Studies of
Texas A&M University
in partial fulfillment of the requirements for the degree of

DOCTOR OF PHILOSOPHY

August 2011

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ABSTRACT

Time-to-Produce, Inventory, and Asset Prices. (August 2011)

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In a production-based general equilibrium model, I study the impact of time-to-build and time-to-produce technology constraints and inventory on asset prices and macroeconomic quantity dynamics. A time-to-build constraint captures the delay in transforming new investment into productive capital; a time-to-produce constraint captures the delay in transforming productive capital into final products. Empirically, I find that the U.S. economy in aggregate exhibits approximately a three-quarter time-to-build and a four-quarter time-to-produce constraint. These delays in the production process introduce short-run risks in the economy where inventory accumulation facilitates consumption smoothing over time. Using this structure for time-to-build and time-to-produce constraints, I numerically calibrate a production-based general equilibrium model where the representative investor has recursive preferences over consumption and inventory. The model delivers first and second moments of macroeconomic quantities and asset prices consistent with the data. A small elasticity of intertemporal substitution is necessary to positively price the short-run risks induced by the production constraints. Inventories help fit the volatilities of asset returns, while the time-to-produce feature ensures nontrivial inventory holdings. In addition, the model is able to match empirical lead-lag patterns between asset prices and macroeconomic quantities as well as observed equity return predictability.

to my wife, Xiaohong, and sons, Tiantian and Tianjian

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TABLE OF CONTENTS

LIST OF TABLES

TABLE Page

LIST OF FIGURES

1. INTRODUCTION

This paper explores the equilibrium impact on asset prices and macroeconomic quantities of production risks in the presence of technology imperfections. In their seminal time-to-build (TTB, hereafter) work, Kydland and Prescott (1982) introduce a technology imperfection in building productive capital and define TTB as the delay in transforming new investment into productive capital. Hence, changes in the current capital stock depend on new projects initiated several periods ago. This paper extends Kydland and Prescott (1982) by incorporating another technology imperfection, namely, the inability to transform productive capital into final goods instantaneously, which I refer to as time-to-produce (TTP, hereafter). TTP is the delay during the transformation from productive capital to final products. With TTP, current output is pre-determined by the productive capital stock chosen several periods ago.

TTP is a natural production friction to consider in several industries, such as agriculture and manufacturing. For example, agricultural crops such as wheat, corn, and soybeans have non-trivial growing seasons between planting and harvesting. In manufacturing, products like the Airbus 380 have extended production times. TTP differs from TTB in several aspects. First, the TTB constraint focuses on the frictions during the formation of productive capital while the TTP constraint focuses on the frictions during the use of productive capital. For example, TTB mainly captures the delays in putting productive capital into place, which includes building plants and installing machines before production, and maintenances during production. In contrast, TTP captures the delays in producing final goods from raw inputs due to technology constraints, e.g., physical, chemical, or capacity constraints. Second, the

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productivity of current capital stock is unobservable under the TTP constraint, while it is measurable under the TTB constraint. Third, the depreciation of productive capital is realized over several periods during production under the TTP constraint.

These delays in production introduced by the TTB and TTP constraints increase short-run consumption risks in the economy. These short-run risks are generated due to three reasons. First, the delays introduce and accumulate uncertainty in the economy. Second, TTB slows down the response of capital stock to productivity shocks, making it more difficult for the agent to use investment to smooth consumption. Third, and most importantly, the agent loses control of output temporarily due to the TTP constraint, thus output will be volatile and so is consumption. These risks are short-run because delays are temporary. As a response, a risk averse agent can employ an inventory technology to smooth her consumption when faced with volatile output. Hence, inventory is the second important ingredient of the model. Empirically, inventory is important at both macro and micro levels. For example, inventory contributes about one-third of the aggregate output volatility, and it is procyclical.[1] At the firm level, total inventory averages about 14% of total assets for COMPUSTAT firms during 3/1984-12/2009, which is much larger than the capital expenditure component. In my model, inventory is a technology with a perfect elasticity of supply and a negative return due to inventory holding costs. To ensure a positive inventory investment, as in the money-in-the-utility function literature (Sidrauski (1967)), I model the demand for inventory via an inventory-in-the-utility specification, because inventory can increase the agent's utility. This can be interpreted as the shopping convenience provided by inventory, e.g., lower shopping costs. Inventory is a quasi-risk-free asset in this economy. The risk-free rate can be defined as the difference between the marginal rate of substitution between consumption

[1]See Fitzgerald (1997) and Hornstein (1998).

and inventory, and the marginal inventory holding cost. Since these two components are positively correlated, i.e., both are counter-cyclical, the risk-free rate can be less volatile. Hence, inventory policy provides one additional dimension to help disentangle the risk-free rate puzzle from the equity premium puzzle in this model.

The third important ingredient of the model is recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Weil (1989)). Recursive utility allows us to differentiate relative risk aversion from the elasticity of intertemporal substitution (EIS, hereafter). This is crucial since in the model, the risks resulting from delays in production are short-lived. A small EIS is necessary to positively price these short-lived risks, i.e., the agent prefers late resolution of uncertainty.[2] Thus the agent prefers to smooth her consumption more over time than across states. Moreover, since the pricing kernel decreases with the inventory growth rate when the EIS is low and inventory is procyclical, the model is able to generate a counter-cyclical pricing kernel when the EIS is low.

I construct a production-based general equilibrium model to study the impact of TTB, TTP, and inventory on equilibrium asset prices and macroeconomic quantities. Assuming $h+1$ periods of TTB and $d+1$ periods of TTP, in addition to the current capital stock, the TTB model uses capital stocks of the future h periods as state variables, while the TTP model uses the historical d period capital stocks as state variables. Because of the additional state variables introduced, the usual equivalence between marginal q and average q (Hayashi (1982), Abel and Eberly (1994)) breaks down.[3] The stock return, R_{t+1}, is connected to the future cash flows up to $h+d+1$

[2]The empirical evidence on the aggregate EIS parameter size is mixed. For example, Attanasio and Weber (1993), Beaudry and van Wincoop (1996), and Vissing-Jorgensen (2002) find a high EIS in disaggregated data. The long-run risk literature (e.g., Bansal and Yaron (2004)) also favors a high EIS. Contrarily, Hall (1988), Campbell and Mankiw (1989), Campbell (2003), Yogo (2004, 2006), Gomes, Kogan, and Yogo (2009), and Beeler and Campbell (2009) find evidence of a small EIS.

[3]Altuğ (1993) and Kuehn (2009) also make this point for the TTB case.

periods. The stock return R_{t+1} in the TTP model is related to not only the current and the next period average q,[4] but also the historical and the future average q, i.e., $(MB_{t-d+1}, ..., MB_t, MB_{t+1}, ..., MB_{t+d+1})$, through the depreciation channel. This is different from the standard production model and the TTB only model. The lengths of TTB and TTP can be identified directly from firm level data, based on the correlations between stock returns and future cash flows, and the correlations between stock returns and average q ratios. Using COMPUSTAT/CRSP data from March 1984 to December 2009, I estimate a three-quarter TTB ($h = 2$) and a four-quarter TTP ($d = 3$) in the U.S. economy.[5]

Computing challenges emerge from the number of state variables and the deeply recursive equilibrium conditions due to the TTB and the TTP constraints when solving the general equilibrium model. I solve the model by a third-order perturbation method (See Judd and Guu (1993, 1997) and Schmitt-Grohé and Uribe (2004)). Quantitatively, the main model can generate macroeconomic quantities and asset prices in line with the data. Besides the macroeconomic quantities often studied in the RBC models, the model is able to match the new variable introduced, inventory. TTP is necessary to induce inventory holdings. Otherwise, inventory demand is negligible since the agent can change output quickly via adjusting the productive capital through investments. Inventory is useful in fitting the volatilities of asset returns. In particular, the model generates a low volatility of the risk-free rate, which has been one of the main challenges in production-based models (Jermann (1998), Boldrin, Christiano, and Fisher (2001), Kuehn (2009), Campanale, Castro, and Clementi (2010), and Kaltenbrunner and Lochstoer (2010)). Additionally, the

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[4]See, e.g., the empirical documentation by Fama and French (1993), the partial equilibrium models of Berk, Green, and Naik (1999) and Zhang (2005), and the general equilibrium cases of Gomes, Kogan, and Zhang (2003) and Kuehn (2007).

[5]Note that Kydland and Prescott (1982) assume a four-quarter TTB, while Zhou (2000) assumes a four-quarter or a six-quarter TTB.

main model produces lead-lag correlations between asset prices and macroeconomic quantities (See Backus, Routledge, and Zin (2007, 2010)), and return predictability (See Cochrane (2008a,b)) observed in the data.

My paper builds on the large literature of production-based general equilibrium asset pricing models (Jermann (1998), Boldrin, Christiano, and Fisher (2001), Kuehn (2007, 2009), Beeler (2009), Guvenen (2009), Campanale, Castro, and Clementi (2010), Croce (2010), and Kaltenbrunner and Lochstoer (2010)). These models typically assume the representative agent is endowed with habit formation or recursive preferences, and introduce risks into the economy through investment frictions or stochastic productivity shocks.[6] Departing from the literature, this paper emphasizes production risks generated by technology frictions (TTB and TTP) instead of investment frictions or exogenously given productivity processes. These technology frictions have been understudied in the asset pricing literature. Only a few papers study the asset pricing implications of TTB. For example, Boldrin, Christiano, and Fisher (2001) investigate TTB under habit formation. Kuehn (2009) studies the interaction between investment returns and equity returns under TTB and CRRA assumptions. This paper adds to the literature by studying TTP in a recursive preferences setting. In particular, this paper quantifies the asset pricing implications of the short-run risks created by these technology frictions.

Recently, Belo (2010) and Jermann (2010) explore the asset pricing implications of producers' first-order conditions in a pure production-based partial equilibrium model. Belo (2010) assumes a firm operates with one type of capital and is able to choose state-contingent productivity levels to smooth output across states. He

[6]Investment frictions include convex capital adjustment costs, investment irreversibility (Kogan (2004)), investment commitment (Kuehn (2007)), and capital immobility (Boldrin, Christiano, and Fisher (2001)). Productivity shocks can be introduced with time-varying volatilities (Beeler (2009)) as well as permanent or transitory components (Kaltenbrunner and Lochstoer (2010) and Croce (2010)).

finds that the production-based pricing kernel estimated from a two-sector economy reasonably captures the cross-section of asset returns. Jermann (2010) assumes a firm operates with as many types of capitals as there are productivity states. He is able to match the equity premium and the risk-free rate observed in the data through highly convex capital adjustment costs and stochastic productivity shocks. Both Belo (2010) and Jermann (2010) emphasize that producers can allocate resources across different productivity states; however, technology frictions emphasized in this paper make such transfers more difficult to achieve. Moreover, the lack of general equilibrium implications makes it unclear whether their models can match other dimensions, such as empirical macroeconomic quantities. In contrast, this paper builds a general equilibrium model to address both macroeconomic quantities and asset returns.

My paper also contributes to the business cycle literature. First, I introduce a TTP constraint and estimate TTB and TTP directly from firm level data. Although TTB has proven to be a source of economic fluctuations, the empirical estimation of TTB is still quite rare and is based mostly on survey data or field studies (Mayer (1960) and Montgomery (1995)). The approach here uses publicly available data, and is applicable at both the aggregate level and the individual firm level. Second, the general equilibrium model provides mechanisms to generate widely observed lead-lag effects, i.e., asset prices lead macroeconomic quantities by about two quarters, which has been a challenge to the standard RBC model in which everything moves together (Barro and King (1984)). Boldrin, Christiano, and Fisher (2001) explain the lead-lag relationship between interest rates and output by assuming labor is determined before observing technology shocks. Gârleanu, Panageas, and Yu (2009) introduce technological innovations into new vintage capital stock, which affect the economy with lags, but asset prices immediately, to create a lead-lag pattern. Backus, Routledge, and Zin (2010) build a long-run risk model, which assumes a positive correlation between consumption growth and stochastic volatility to capture such cross-correlations. This paper constructs a production-based general equilibrium model with production delays to endogenize such lead-lag patterns. Since new investments generate final goods with lags but affect asset prices immediately, asset prices lead macro quantities. Quantitatively, I find that TTB pins down the length of the lead-lag relationship while the TTP helps match the magnitude of the correlations.

My paper is related to the small but growing literature on inventory, goods durability, and asset pricing. The most closely related works include Gomes, Kogan, and Yogo (2009), Belo and Lin (2009), and Jones and Tuzel (2010). Following the inventory literature,[7] these papers model inventory from a production perspective, using it either as a factor input or stockout avoidance, together with convex or non-convex (S, s) inventory adjustment costs. Specifically, Gomes, Kogan, and Yogo (2009) use inventory as the key feature to distinguish durable and nondurable goods, and model inventory as a factor input in the durable good production. Belo and Lin (2009) document that firms with lower inventory growth rates earn about a 7% per year higher return. Such a finding is difficult to explain with a partial equilibrium model with production-motivated inventory. Jones and Tuzel (2010) also develop a partial equilibrium model to study the response of inventory to the changes in the cost of capital. My model departs from this literature along four dimensions. First, I investigate inventory from the general equilibrium perspective, in particular, the consumption smoothing role of inventory. This is different from the traditional production-motivated models. Second, I model the total inventory holding costs. I find that inventory holding costs are concave in inventory in the macroeconomic data, which implies that inventory holding costs are marginally diminishing. This is

[7] An incomplete list of inventory models includes: production smoothing motivations (Eichenbaum (1989), Blinder and Maccini (1991), and Ramey and West (1999)), inventory as an exogenous input factor in production (Kydland and Prescott (1982), Christiano (1988), Jones and Tuzel (2010), and Gomes, Kogan, and Yogo (2009)), stockout avoidance (Bils and Kahn (2000) and Ramey and West (1999)), and the (S, s) rule (Khan and Thomas (2007)).

different from the convex or non-convex (S,s) inventory adjustment costs used in the literature. Third, although delays in production might be related to good durability since durable good production likely has longer delays, they are different as the latter emphasizes the long-lasting consumption and utility consequences of a durable good while the former emphasizes production frictions. Fourth, inventory is durable in my model and it can be instantly transformed into a non-durable consumption good, while Gomes, Kogan, and Yogo (2009) use it as a factor input in the durable good production.

The paper proceeds as follows. I first construct a production-based general equilibrium model in Section 2. Then, I estimate the lengths of TTB and TTP from firm-level data in Section 3. Section 4 describes computing challenges specific to this model and the numerical solution. Section 5 outlines the empirical data and parameters used in the calibrations. Section 5.4 presents the main numerical results. Finally, Section 6 concludes.

2. A GENERAL EQUILIBRIUM MODEL

Consider an all-equity representative firm, which produces one real good and operates in a discrete and infinite time horizon. This abstracts from the complications of real world production, which features different goods and multiple levels of intermediate goods production. Multi-stage production can be viewed as a production chain of multiple firms considered here, in the sense of Levine (2011).[1] Uncertainty in the economy, the aggregate productivity shock, is represented by a filtered probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_s\}_{s\geq 0}, \mathbb{P})$, which satisfies the usual conditions.

2.1 Firms

The representative firm only uses productive capital to produce one good,[2]

$$y_t = K_{t-d}^{\alpha} Z_t^{1-\alpha}, \tag{2.1}$$

where y_t is output at time t, Z_t is an aggregate productivity shock at time t, and K_{t-d} is the capital stock at the beginning of time $t-d$. Here, d denotes the time delay in production, capturing a TTP constraint. That is, the output at time t depends on the production started at time $t-d$. So the outputs in the next d periods are uncontrollable and predetermined by the historical capital stock levels. Especially, the productivity of current capital stock is unobservable since the output will be realized d periods later. By setting $d = 0$, a conventional production model is recovered.

The aggregate productivity shock follows an AR(1) process,

$$z_{t+1} = \rho\, z_t + \sigma\, \varepsilon_{t+1}, \tag{2.2}$$

[1]It is necessary to define the boundary of firms and the input-output structure of the economy to incorporate intermediate goods production, which is beyond the scope of this paper.

[2]That is, the labor input is normalized to 1.

where $z_t = log\, Z_t$, $0 < \rho < 1$, ε_{t+1} is a standard normal distribution, and σ is a scaling factor on the aggregate productivity shock. Clearly, the productivity shock is transitory.

The firm problem also incorporates a TTB constraint which impacts the capital evolution of the firm. Following the TTB literature (Kydland and Prescott (1982)), I assume there is a time delay of $h + 1$ periods in building productive capital. Let the motion of capital stock be

$$K_{t+1} = K_t + g_t - \delta \sum_{i=0}^{d} K_{t-i} u_i, \tag{2.3}$$

and

$$g_t = g(S_{t-h}, K_{t-d}), \tag{2.4}$$

where g_t is the capital formation function, S_{t-h} is the project size initiated at time $t - h$, and δ is the depreciation rate. The productive capital is assumed to be depreciated through $d + 1$ periods with weight u_i at period i where $\sum_{i=0}^{d} u_i = 1$. If $u_0 = 1$, then depreciation occurs only when output is finished. Similarly, it is fully depreciated when the production begins if $u_d = 1$. We obtain the standard firm problem when $h = 0$ and $d = 0$, and the case of Kydland and Prescott (1982) when $h = 3$ and $d = 0$.

The capital formation function, g_t, is specified as in Jermann (1998), i.e.,

$$g_t = g(S_{t-h}, K_{t-d}) = \zeta \left(\frac{S_{t-h}}{K_{t-d}} \right) K_{t-d} = \left[\frac{a_1}{1 - 1/\chi} \left(\frac{S_{t-h}}{K_{t-d}} \right)^{1-1/\chi} + a_2 \right] K_{t-d}, \tag{2.5}$$

where χ governs the capital adjustment costs, and a_1 and a_2 are constants. The capital adjustment costs are high when χ is low, and there is no capital adjustment costs when $\chi \to \infty$. As in Boldrin, Christiano, and Fisher (2001), the constants a_1

and a_2 are chosen so that there is no capital adjustment cost in the deterministic steady state.[3] Hence, a_1 and a_2 are set as

$$\begin{aligned} a_1 &= \delta^{1/\chi}, \\ a_2 &= \frac{1}{1-\chi}\delta. \end{aligned}$$

The total investment at time t, I_t, is

$$I_t = \sum_{i=0}^{h} w_i S_{t-i}, \tag{2.6}$$

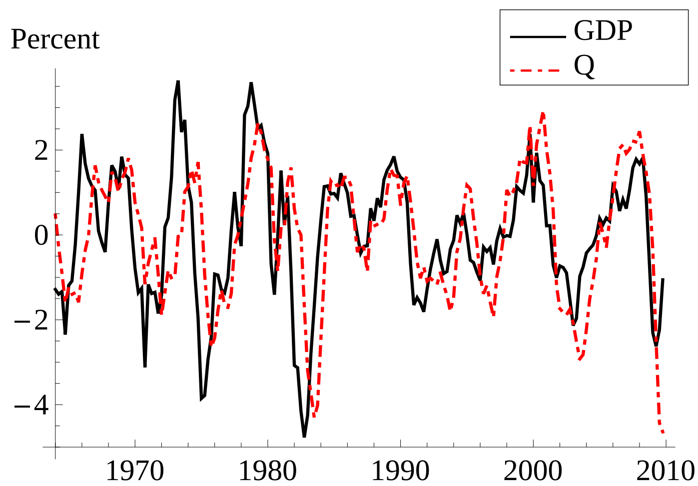
where w_i is the investment expenditure weight of the project initiated at time $t-i$ with $\sum_{i=0}^{h} w_i = 1$.

2.2 Households

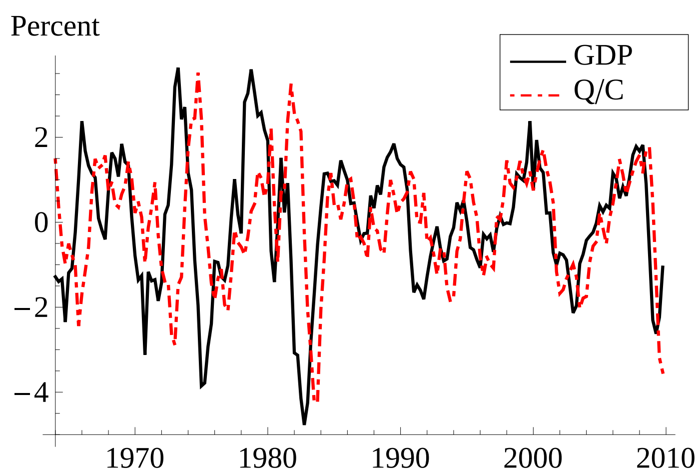
Due to the delays in production, the representative agent uses inventory to smooth consumption when facing fluctuating output. This implies that inventory holdings move with output. Empirically, inventory holdings and GDP move together, with inventory holdings lagging aggregate output slightly, as shown in Figure 2.1(a). The correlation between the cyclical component of inventory and that of GDP is 0.58. The fact that inventory is procyclical confirms the consumption smoothing role of inventory.

Inventory in this model is narrowly interpreted as the inventory of final goods, without referring to raw materials and work-in-process. Although there is no depreciation in inventory given the real good assumption, the agent pays storage costs

[3] In the steady state, $\zeta\left(\frac{S}{K}\right) = \frac{S}{K}$ and $\zeta'\left(\frac{S}{K}\right) = 1$, where S and K are the project and capital stock at the steady state.



(a) Cyclical Component of Real GDP and Inventory Holdings



(b) Cyclical Component of Real GDP and Inventory/Consumption Ratio

Fig. 2.1.: Cyclical Components.

Figure (a) and (b) plot the cyclical component of real GDP, inventory holdings (Q), and inventory/cosumption ratio (Q/C) over 1964-2009, computed from the Hodrick-Prescott filter. Quarterly data from the NIPA tables are used.

and also faces inventory risk. For tractability, the inventory holding cost is specified similar to the capital adjustment cost function. At time t, the inventory cost h_t is

$$h_t = h(Q_t, K_{t-d}) = \frac{\eta}{\tau}\left(\frac{Q_t}{K_{t-d}}\right)^{\tau} K_{t-d}, \tag{2.7}$$

where Q_t is the inventory level at the end of time t, τ is the curvature parameter, and η is the coefficient of inventory cost. So, the inventory holding cost is modeled as homogeneous of degree one in inventory and the productive capital stock. It is concave in inventory when $\tau < 1$, which implies a marginally decreasing inventory holding cost. Inventory holding cost h_t is captured as a proportion of productive capital since output depends on productive capital stock.

To complete the modeling of inventory, a benefit of inventory needs to be specified to offset the inventory cost. Otherwise, the optimal inventory demand will always be 0. Generally, the benefit of inventory can be modeled through either the production technology (Kydland and Prescott (1982), Christiano (1988), Gomes, Kogan, and Yogo (2009), Belo and Lin (2009), and Jones and Tuzel (2010)) or the utility function directly (Kahn, McConnell, and Perez-Quiros (2002) and Iacoviello, Schiantarelli, and Schuh (2010)). While it would be ideal to directly model the microeconomic frictions that drive the benefits of holding inventory from a firm's perspective, I instead take a reduced form approach by modeling the benefit of holding inventory through an inventory-in-the-utility specification for three reasons. First, here inventory is used as a consumption smoothing device when the agent faces volatile output, rather than a factor input in production. The economic interpretation is that inventory can increase utility by providing shopping convenience, security benefits and lowering transaction costs. Second, in a one-firm and one-agent setting, modeling the benefits of holding inventory through the agent's utility is equivalent to modeling it from a firm's perspective. Third, modeling inventory through utility provides an analytical advantage to study the asset pricing implications of the model.

Borrowing from the money-in-the-utility function literature (Sidrauski (1967)), I adopt an inventory-in-the-utility function specification given by

$$U_t = \left\{(1-\beta)\left[\upsilon C_t^{\omega} + (1-\upsilon)Q_t^{\omega}\right]^{\frac{1-\gamma}{\omega\theta}} + \beta[\mathbb{E}_t U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \tag{2.8}$$

where C_t is the consumption at time t, β is the time discount, υ is the consumption share with $0 < \upsilon < 1$, γ measures the relative risk aversion, $\omega = 1 - \frac{1}{\phi}$, ϕ is the elasticity of substitution between inventory and consumption, $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, and ψ is the elasticity of intertemporal substitution. Clearly, when $\phi = \psi$, the utility is additively separable of inventory and consumption. It is a Cobb-Douglas specification when $\phi = 1$. Similar non-separable utility has been widely used in the literature (e.g., Piazzesi, Schneider, and Tuzel (2007), Yogo (2006), Gomes, Kogan, and Yogo (2009), and Uhlig (2009)). This utility specification can be viewed as inventory-augmented recursive preferences (Kreps and Porteus (1978), Epstein and Zin (1989, 1991), and Weil (1989)). This specification implies that inventory is durable and it can be instantly transformed into the non-durable consumption good, which is different from Gomes, Kogan, and Yogo (2009) where inventory is used as a factor input in the durable good production.

It is straightforward to see that the pricing kernel is

$$M_{t,t+1} = \beta\left[\frac{C_{t+1}}{C_t}\right]^{-\frac{1}{\psi}}\left[\frac{\upsilon + (1-\upsilon)\left(\frac{Q_{t+1}}{C_{t+1}}\right)^{1-\frac{1}{\phi}}}{\upsilon + (1-\upsilon)\left(\frac{Q_t}{C_t}\right)^{1-\frac{1}{\phi}}}\right]^{\frac{\frac{1}{\phi}-\frac{1}{\psi}}{1-\frac{1}{\phi}}}\left[\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t U_{t+1}^{1-\gamma}}\right]^{1-\frac{1}{\theta}}. \tag{2.9}$$

There are three components in the pricing kernel, namely, the growth rate of consumption, the growth rate of inventory/consumption ratio, and the forward looking part due to the recursive preferences. The last term disappears for separable utility (i.e., $\gamma = \frac{1}{\psi}$). Clearly, inventory directly enters the pricing kernel because of the inventory-in-the-utility function specification when $\phi \neq \psi$. Empirically, as shown in Figure 2.1(b), the inventory/consumption ratio is positively related to aggregate output with a correlation of 0.16. So, the procyclical inventory/consumption ratio aids in generating a counter-cyclical pricing kernel when $\psi < \phi$.[4] This is a preference constraint implied by asset prices in this model.

The representative agent owns and runs the firms. She consumes all dividends paid by the firms. That is,

$$C_t = d_t = y_t - h_t - I_t - (Q_t - Q_{t-1}). \tag{2.10}$$

Therefore, the equity return is

$$R_{t+1} = \frac{P_{t+1} + C_{t+1}}{P_t} \tag{2.11}$$

where P_t is the stock price at time t.

The representative agent chooses the optimal inventory and consumption profiles, while the firm operates under its optimal investment policy. The agent's problem can be summarized as follows:

$$\begin{aligned}
U_t &= U(K_{t-d}, ..., K_t, S_{t-h}, ..., S_{t-1}, Q_{t-1}, Z_t) \\
&= \max_{\{C_t, S_t, K_{t+1}, Q_t\}} \left\{ (1-\beta) \left[\upsilon C_t^{\omega} + (1-\upsilon) Q_t^{\omega} \right]^{\frac{1-\gamma}{\omega\theta}} + \beta [\mathbb{E}_t U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}} \\
s.t. \quad & C_t = K_{t-d}^{\alpha} Z_t^{1-\alpha} - \sum_{i=0}^{h} w_i S_{t-i} - h_t - (Q_t - Q_{t-1}), \qquad (2.12) \\
& K_{t+1} = K_t + g_t - \delta \sum_{i=0}^{d} u_i K_{t-i}. \qquad (2.13)
\end{aligned}$$

To distinguish from the standard production model, the TTB only model, and a model with TTB and TTP, it is instructive to look at the state variables in these models. The standard production model uses current capital stock, the productivity shock, and inventory at the end of previous period, $\{K_t, Z_t, Q_{t-1}\}$, as state variables. The TTB constraint expands the state space by introducing historical initiated projects, i.e., $\{S_{t-h}, ..., S_{t-1}, K_t, Z_t, Q_{t-1}\}$. Given the capital stock dynamics, this is

[4] Yogo (2006) also makes similar observations.

equivalent to $\{K_t, K_{t+1}, ..., K_{t+h}, Z_t, Q_{t-1}\}$. Incorporating the TTP constraint in addition to the TTB constraint adds historical capital stocks to the state space, i.e., $\{S_{t-h}, ..., S_{t-1}, K_{t-d}, ..., K_{t-1}, K_t, Z_t, Q_{t-1}\}$. Again, by the motion of capital stock, this can be rewritten as $\{K_{t-d}, ..., K_{t-1}, K_t, K_{t+1}, ..., K_{t+h}, Z_t, Q_{t-1}\}$.

2.3 The Equilibrium Conditions

The Lagrangian function of the maximization problem is

$$\begin{aligned} L_t &= \left\{(1-\beta)\left[\upsilon C_t^{\omega} + (1-\upsilon)Q_t^{\omega}\right]^{\frac{1-\gamma}{\omega\theta}} + \beta\left[\mathbb{E}_t U_{t+1}^{1-\gamma}\right]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \\ &\quad +\mu_t\left[K_{t-d}^{\alpha} Z_t^{1-\alpha} - \sum_{i=0}^{h} w_i S_{t-i} - h_t - (Q_t - Q_{t-1}) - C_t\right] \\ &\quad +\xi_t\left[K_t + g_t - \delta\sum_{i=0}^{d} u_i K_{t-i} - K_{t+1}\right], \end{aligned} \tag{2.14}$$

where $\{\mu_t, \xi_t\}$ are the current value Lagrangian multipliers associated with constraints (2.12)-(2.13), respectively.

From the first order condition with respect to C_t at time t, we have the following condition:

$$\mu_t = (1-\beta)\upsilon U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}} \left[\upsilon + (1-\upsilon)\left(\frac{Q_t}{C_t}\right)^{1-\frac{1}{\phi}}\right]^{\frac{\frac{1}{\phi}-\frac{1}{\psi}}{1-\frac{1}{\phi}}}. \tag{2.15}$$

This defines the marginal utility of consuming one additional unit of good at time t. Similar to the pricing kernel, marginal utility decreases with inventory/consumption ratio when $\psi < \phi$. Hence, a small EIS is necessary to make the procyclical inventory/consumption ratio generate a counter-cyclical marginal utility, and in the end, a counter-cyclical pricing kernel, as seen in (2.9).

Similarly, the optimal inventory policy satisfies the following first order condition:

$$\begin{aligned} \frac{\partial L_t}{\partial Q_t} = 0 &= (1-\beta)(1-\upsilon)U_t^{\frac{1}{\psi}} Q_t^{\omega-1}\left[\upsilon C_t^{\omega} + (1-\upsilon)Q_t^{\omega}\right]^{\frac{1-\gamma}{\omega\theta}-1} \\ &\quad +\mathbb{E}_t\left[\frac{\partial U_t}{\partial U_{t+1}}\frac{\partial U_{t+1}}{\partial Q_t}\right] - \mu_t\left[\frac{\partial h_t}{\partial Q_t} + 1\right]. \end{aligned} \tag{2.16}$$

From the envelope condition, we have

$$\frac{\partial U_{t+1}}{\partial Q_t} = \frac{\partial L_{t+1}}{\partial Q_t} = \mu_{t+1}. \tag{2.17}$$

Applying the envelope condition to (2.16), we obtain

$$\mathbb{E}_t[M_{t,t+1}] = -\frac{1-\upsilon}{\upsilon}\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}} + \frac{\partial h_t}{\partial Q_t} + 1. \tag{2.18}$$

This equation captures the mean of the stochastic discount factor, and indeed, the risk-free rate. The risk-free rate can be approximated as:

$$r_{f,t} \approx \frac{1-\upsilon}{\upsilon}\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}} - \frac{\partial h_t}{\partial Q_t} = \frac{1-\upsilon}{\upsilon}\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}} - \eta\left(\frac{Q_t}{K_{t-d}}\right)^{\tau-1}. \tag{2.19}$$

The first item on the right hand side is the marginal rate of substitution between consumption and inventory, while the other items on the right hand side capture the marginal cost of increasing one additional unit of inventory at time t. Intuitively, inventory is a quasi-risk-free asset in the economy, so the risk-free rate is jointly determined by the optimal consumption and inventory choices. This is important, as the inventory policy gives us one additional dimension to disentangle the risk-free rate from the risky asset returns. Thus, the tight connection between the risk-free rate puzzle and the equity premium puzzle breaks down here. The volatility of the risk-free rate can be written as

$$\begin{aligned} Var(r_{f,t}) \;\approx\; & \left(\frac{1-\upsilon}{\upsilon}\right)^2 Var\left[\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}}\right] + \eta^2 Var\left[\left(\frac{Q_t}{K_{t-d}}\right)^{\tau-1}\right] \\ & -2\eta\frac{1-\upsilon}{\upsilon}Cov\left[\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}}, \left(\frac{Q_t}{K_{t-d}}\right)^{\tau-1}\right]. \end{aligned} \tag{2.20}$$

Hence, to generate a less volatile risk-free rate, we need

$$Cov\left[\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}}, \left(\frac{Q_t}{K_{t-d}}\right)^{\tau-1}\right] > 0. \tag{2.21}$$

That is, the marginal rate of substitution between consumption and inventory and the marginal inventory holding cost should be positively correlated. Since the consumption/inventory ratio is counter-cyclical and the inventory/capital ratio is procyclical in the data, this requires $\tau < 1$. So, this is a technology constraint implied by a low volatility risk free rate. In this case, both the marginal rate of substitution between consumption and inventory and the marginal inventory holding cost are counter-cyclical. Intuitively, in good (bad) times, inventory is relatively cheap (expensive), so although both consumption and inventory increase (decrease), inventory increases (decreases) more. Since changes in the consumption/inventory ratio are partly offset by changes in the marginal inventory cost, the risk-free rate can be less volatile.

The optimal capital stock at time $t+1$ satisfies

$$\frac{\partial L_t}{\partial K_{t+1}} = \mathbb{E}_t \left[\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial K_{t+1}} \right] - \xi_t = 0, \tag{2.22}$$

where ξ_t is the marginal utility at time t of increasing one additional unit of capital stock at time $t+1$. Therefore, $\frac{\xi_t}{\mu_t}$ is the marginal q usually defined in the production model. For simplicity, define

$$q_t \equiv \frac{\xi_t}{\mu_t}. \tag{2.23}$$

Applying the envelope conditions recursively, we obtain the evolution of marginal q as follows,

$$\begin{aligned} q_t \;\; = \;\; & \mathbb{E}_t \left[M_{t,t+1}\, q_{t+1} \right] - \delta \mathbb{E}_t \left[\sum_{i=1}^{d+1} M_{t,t+i}\, u_{i-1}\, q_{t+i} \right] + \\ & \mathbb{E}_t \left\{ M_{t,t+d+1} \left[\alpha K_{t+1}^{\alpha-1} \left(Z_{t+d+1} \right)^{1-\alpha} + q_{t+d+1} \frac{\partial g_{t+d+1}}{\partial K_{t+1}} - \frac{\partial h_{t+d+1}}{\partial K_{t+1}} \right] \right\}. \end{aligned} \tag{2.24}$$

We see that current q is linked to the future q, $\{q_{t+1}, ..., q_{t+d+1}\}$ through the depreciation channel. This property originates from the TTP feature only. Both the standard production model and the TTB model can not provide this feature.

The optimal investment policy satisfies the following first order condition,

$$\frac{\partial L_t}{\partial S_t} = \mathbb{E}_t \left[\frac{\partial U_t}{\partial U_{t+1}} \frac{\partial U_{t+1}}{\partial S_t} \right] - \mu_t w_0 = 0. \tag{2.25}$$

Applying the envelope conditions recursively, the above condition can be simplified to

$$\mathbb{E}_t \left[M_{t,t+h}\, q_{t+h} \frac{\partial g_{t+h}}{\partial S_t} \right] = \mathbb{E}_t \left[\sum_{i=0}^{h} M_{t,t+i} w_i \right]. \tag{2.26}$$

Therefore, the above condition presents the marginal benefit and cost at time t of adding one additional unit of capital stock at time $t+h+1$.

2.4 Stock Returns and Marginal q

Let firm value be $V_t = P_t + C_t$. From (2.24), the investment return, R^I_{t+1}, can be written in terms of marginal q as follows

$$\begin{aligned} R^I_{t+1} \;&=\; \frac{\frac{\partial V_{t+1}}{\partial K_{t+1}}}{q_t} = \frac{1}{q_t}\Bigg\{ q_{t+1} - \delta \mathbb{E}_{t+1}\left[\sum_{i=1}^{d+1} M_{t+1,t+i}\, u_{i-1}\, q_{t+i}\right] + \mathbb{E}_{t+1}\Bigg[\\ & M_{t+1,t+d+1}\left(\alpha K^{\alpha-1}_{t+1} Z^{1-\alpha}_{t+d+1} + q_{t+d+1}\frac{\partial g_{t+d+1}}{\partial K_{t+1}} - \frac{\partial h_{t+d+1}}{\partial K_{t+1}}\right)\Bigg]\Bigg\} \end{aligned} \quad (2.27)$$

and satisfies $\mathbb{E}_t[M_{t,t+1}R^I_{t+1}] = 1$. Moreover, the above equation implies that firm value at time $t+1$, V_{t+1}, is related to $q_{t+1}, ..., q_{t+d+1}$. Again, this is due to the fact that the historical capital stocks are in the state space now. Similarly, the stock price at time t, P_t, is related to $q_t, ..., q_{t+d}$.

By definition, the stock return R_{t+1} is

$$\begin{aligned} R_{t+1} = \frac{V_{t+1}}{P_t} \;&=\; \frac{V_{t+1}}{\mathbb{E}_t M_{t,t+1}[V_{t+1} - \frac{\partial V_{t+1}}{\partial K_{t+1}} K_{t+1}] + q_t K_{t+1}} \\ &=\; \frac{V_{t+1}/K_{t+1}}{\mathbb{E}_t M_{t,t+1}[V_{t+1}/K_{t+1} - \frac{\partial V_{t+1}}{\partial K_{t+1}}] + q_t}. \end{aligned} \quad (2.28)$$

Hence, the stock return does not equal the investment return since $\frac{V_{t+1}}{K_{t+1}} \neq \frac{\partial V_{t+1}}{\partial K_{t+1}}$ in this case.

Since marginal q is unobservable, we instead compute the average q as

$$MB_t = \frac{P_t}{K_{t+1}} = q_t + \mathbb{E}_t M_{t,t+1}\left[\frac{V_{t+1}}{K_{t+1}} - \frac{\partial V_{t+1}}{\partial K_{t+1}}\right]. \quad (2.29)$$

Similarly, average q and marginal q are usually different. This is in contrast to Hayashi (1982) and Abel and Eberly (1994), where they show the equivalence of average q and marginal q under homogeneous assumptions. Clearly, the equivalence holds only when there is no technology imperfections. Additionally, since P_t is related to $q_t, ..., q_{t+d}$, we see the average q, MB_t, is related to $q_t, ..., q_{t+d}$ as well.

Substituting (2.29) into (2.28), the stock return can be rewritten as

$$R_{t+1} = \frac{V_{t+1}}{K_{t+1} MB_t}.$$

Since V_{t+1} is related to $q_{t+1}, ..., q_{t+d+1}$ and MB_t can be represented by $q_t, ..., q_{t+d}$, we see that the stock return R_{t+1} is not only mechanically related to (MB_t, MB_{t+1}), but also related to both historical and future average q, i.e., $(MB_{t-d+1}, ..., MB_t, MB_{t+1}, ..., MB_{t+d+1})$. The connection between stock returns and historical average q arises from the depreciation channel of the TTP model.

3. ESTIMATING TTB AND TTP

In this section, I estimate the lengths of TTB and TTP from firm level data to guide the specification in the calibrations.

3.1 Empirical Regression

From the previous section, we saw that the correlations between stock returns and historical and future average q tell us the length of TTP. Yet, these average q ratios are not sufficient statistics for firm value. We know that firm value V_{t+1} should also include predetermined future net outputs up to $h+d+1$ periods, which result from unfinished production and partly installed projects because of the TTB and the TTP constraints. Additionally, there are continued investment expenditures up to h periods for unfinished investment projects initiated in the previous periods, which should be excluded from firm value. Last, inventories inherited from the previous period also contribute to firm value. Hence, the correlations between stock returns, average q ratios, future net income, investment expenditure, and inventory give us a way to estimate the lengths of TTB and TTP. Summarizing, the empirical regression is[1]

$$\begin{aligned} R_{t,t+1} \quad = \quad & \alpha + \sum_{i=1}^{d-1} \beta_{-i} MB_{t-i} + \beta_0 MB_t + \sum_{i=1}^{d+1} \beta_i MB_{t+i} + \beta_{size} Log(Size_t) \\ & + \beta_Q Inventory_t + \sum_{i=1}^{h} \beta_{I,i} Investment_{t+i} + \sum_{i=1}^{h+d+1} \beta_{NI,i} NetIncome_{t+i} \\ & + \beta_{IK} I_t/K_t + \varepsilon_{t+1}, \end{aligned} \tag{3.1}$$

where $R_{t,t+1}$ is the quarterly stock return from time t to $t+1$, MB is the ratio of market equity to book equity, $Size_t$ is the firm size, $Inventory_t$ is the inventory at

[1]This empirical regression can be rigorously derived from a partial equilibrium producers' problem.

the end of time t normalized by $Size_t$, $Investment$ is the investment expenditure for projects initiated in previous h quarters normalized by $Size_t$, $NetIncome$ is the cash flow in the future $h+d+1$ quarters normalized by $Size_t$, and I/K is the ratio of investment to capital which has been found to be important in explaining asset returns (e.g., Chen, Novy-Marx, and Zhang (2010)). I employ the standard Fama and MacBeth (1973) cross sectional regression to estimate the market wide delays.

3.2 Data

Subject to data availability, the sample is from March 1984 to December 2009. Only common stocks (share codes 10 or 11) on the NYSE/AMEX/NASDAQ are included. I exclude all financial (SIC code between 6000 and 6999) and utility firms (SIC code between 4900 and 4999), and firms with fiscal quarters ending on a month other than March/June/September/December. The quarterly stock returns are computed from monthly stock returns from CRSP. Financial data are obtained from the COMPUSTAT quarterly files. Specifically, the ratio of market equity to book equity, MB, is computed as in Fama and French (1993), $Size_t$ is the market capitalization at the end of a quarter, $Inventory_t$ is from COMPUSTAT Item INVTQ at time t normalized by $Size_t$.[2] $Investment$ is computed from COMPUSTAT Item CAPXY normalized by $Size_t$,[3] $NetIncome$ is the net income available to common equities (Item IBCOMQ) normalized by $Size_t$, and I/K is the ratio of investment to total assets (Item ATQ). I assume a 3-month delay in financial reporting in order to match financial data with stock price data.

[2]Cash might play a role similar to inventory. For example, Bates, Kahle, and Stulz (2009) find inventory has been used by firms for precautionary purpose. I have also measured inventory with cash, and the results are similar.

[3]Since the detail investment plans are unavailable, here I use the total investment as a proxy for the investment expenditure of projects initiated previously.

Table 3.1: Summary Statistics.

This table summarizes the quarterly financial data and stock returns during 3/1984-12/2009. Only common stocks (share codes 10 or 11) in the NYSE/AMEX/NASDAQ are included, excluding all financial (SIC code between 6000 and 6999) and utility firms (SIC code between 4900 and 4999) and firms with fiscal quarters ending on a month other than March/June/September/December. The quarterly stock returns are computed from the monthly stock returns from CRSP. The total assets is Compustat quarterly item ATQ. The book equity is computed as in Fama and French (1993), and firm size is the market capitalization at the end of a fiscal quarter. The capital expenditure is computed from the year-to-date capital expenditure (Compustat Item CAPXY). The total inventory is from Compustat Item INVTQ. Net income is the income before extraordinary items available for common shareholders (Compustat Item IBCOMQ).

Variable	N	Mean	Median	Min	Max	Std Dev
Quarterly Returns (%)	328511	3.47	0.00	-97.79	1833.18	37.35
Total Assets (MM$)	336672	1584.83	101.49	0.13	846988.00	11954.23
Book Equity (MM$)	336672	625.14	53.29	0.00	169652.00	3764.04
Firm Size (MM$)	335071	1653.80	106.59	0.00	604414.75	10779.52
Book Equity/Market Equity	335060	0.88	0.50	0.00	58070.59	100.35
Investments						
Capital Expenditures (MM$)	322178	24.92	1.09	-2285.00	9679.00	169.60
Capital Expenditures/Total Assets (%)	322178	1.74	0.97	-738.85	775.31	3.70
Inventory						
Inventory (MM$)	330367	127.41	6.45	0.00	85659.00	752.70
Inventory/Total Assets (%)	330367	13.99	9.73	0.00	97.19	15.18
Net Income						
Net Income (MM$)	335751	15.12	0.46	-44905.00	22625.00	233.00
Net Income/Firm Size (%)	334157	-2.93	0.75	-53955.56	37647.06	122.77

3.3 Empirical Results

Table 3.1 presents the main variables used in the regressions. The mean quarterly return is 3.47% with a standard deviation of 37.35%. The mean book-to-market equity is 0.88 with a median of 0.50. The median capital expenditure is 1.09 million dollars, and the median ratio of investment to total assets is 0.97%. Surprisingly, inventory is much larger than investment. The median inventory is 6.45 million dollars, and the median ratio of inventory to total assets is 9.73%.

3.3.1 Market-wide Delays

To apply (3.1) as a regression, we need to assume the maximum time delays first. Since the above regression requires $h + 2d + 1$ consecutive quarterly observations, to balance the number of sample firms and the estimation precision as well as given the 4-quarter TTB assumption in Kydland and Prescott (1982), I assume there are no more than 5 quarters of market-wide delays in building productive capital and producing the final good, i.e., $h = 4$ and $d = 4$.[4]

The cross-sectional regression results are reported in Table 3.2. Starting with Panel A, which includes all sample firms, the first two regressions (Model (1) and (2)) confirm the well documented size and market-to-book effects. The third regression examines the correlation between previous, current, and future market equity-to-book equity ratios and stock returns. The stock returns are positively correlated with future market-to-book ratios (up to time $t + 3$) while negatively related to historical market-to-book ratios (up to time $t - 2$), with an average R^2 of 0.116. Although the evidence from historical and future market-to-book ratios is not quite symmetric, Model (3) suggests $d = 2 \sim 3$. Model (4) adds size and inventory to the regressors. Not surprising, inventory is positively related to expected stock returns. Model (5) is a fully specified regression, including future net income and investment expenditure. The current stock returns are positively correlated with future net income up to time $t + 5$. This suggests that $h + d = 4$. In Model (6), the ratio of investment to capital is added to the regression. Although there is a negative relation between the ratio of investment to capital and stock returns, the results for the other regressors are largely the same.

To see the difference between manufacturing and non-manufacturing firms, I restrict the sample to manufacturing firms (with SIC code between 1000 and 4000)

[4] I also experimented with different maximum h and d values and the results are similar.

Table 3.2: Estimating Time-to-Build and Time-to-Produce: Fama-MacBeth Regressions.

This table estimates time-to-build (h) and time-to-produce (d), from the following cross-sectional regression:

$$\begin{aligned} R_{t,t+1} &= \alpha + \sum_{i=1}^{d-1} \beta_{-i} MB_{t-i} + \beta_0 MB_t + \sum_{i=1}^{d+1} \beta_i MB_{t+i} + \beta_{size} Log(Size_t) \\ &\quad + \beta_Q Inventory_t + \sum_{i=1}^{h} \beta_{I,i} Investment_{t+i} + \sum_{i=1}^{h+d+1} \beta_{NI,i} NetIncome_{t+i} \\ &\quad + \beta_{IK} I_t/K_t + \varepsilon_{t+1} \end{aligned}$$

where $R_{t,t+1}$ is the quarterly stock return from time t to $t+1$, MB is the ratio of market equity to book equity, computed as in Fama and French (1993), $Size_t$ is the market capitalization, $Inventory_t$ is from Compustat Item INVTQ normalized by $Size_t$, $Investment$ is computed from Compustat Item CAPXY normalized by $Size_t$, $NetIncome$ is the net income available to common equities (Item IBCOMQ) normalized by $Size_t$, and I/K is the ratio of investment to total assets (Item ATQ). Only common stocks (share codes 10 or 11) in the NYSE/AMEX/NASDAQ are included, excluding all financial (SIC code between 6000 and 6999) and utility firms (SIC code between 4900 and 4999) and firms with fiscal quarters ending on a month other than March/June/September/December. All coefficients are in percentage and t statistics are reported in parentheses. The sample period is 3/1984-12/2009.

Table 3.2: Continued.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel A All Individual Stocks						
$Log(Size_t)$	-0.77			-0.74	-1.01	-1.00
	(-4.01)			(-4.05)	(-6.20)	(-6.18)
MB_{t-3}			-0.06	-0.05	-0.05	-0.05
			(-1.18)	(-1.00)	(-1.18)	(-1.36)
MB_{t-2}			-0.35	-0.32	-0.28	-0.28
			(-3.61)	(-3.46)	(-3.81)	(-3.75)
MB_{t-1}			-0.28	-0.26	-0.27	-0.26
			(-3.73)	(-3.60)	(-3.79)	(-3.66)
MB_t		-0.10	-2.24	-2.17	-2.13	-2.13
		(-1.89)	(-6.34)	(-6.21)	(-6.40)	(-6.38)
MB_{t+1}			2.72	2.71	2.70	2.70
			(7.46)	(7.41)	(7.60)	(7.57)
MB_{t+2}			0.64	0.65	0.62	0.63
			(5.35)	(5.31)	(5.27)	(5.37)
MB_{t+3}			0.18	0.19	0.17	0.17
			(2.06)	(2.11)	(1.77)	(1.74)
MB_{t+4}			0.05	0.08	0.14	0.13
			(0.83)	(1.34)	(2.33)	(2.23)
MB_{t+5}			-0.01	-0.02	-0.00	0.00
			(-0.45)	(-0.69)	(-0.11)	(0.04)
$Inventory_t$				1.44	1.53	1.38
				(5.16)	(5.32)	(4.53)
$NetIncome_{t+1}$					17.86	17.96
					(7.01)	(7.06)
$NetIncome_{t+2}$					25.41	25.37
					(9.05)	(9.10)
$NetIncome_{t+3}$					12.96	12.87
					(4.74)	(4.70)
$NetIncome_{t+4}$					10.31	10.29
					(5.20)	(5.20)
$NetIncome_{t+5}$					6.83	6.80
					(3.94)	(3.94)
$NetIncome_{t+6}$					1.55	1.73
					(0.86)	(0.94)
$NetIncome_{t+7}$					2.24	2.09
					(1.12)	(1.12)
$NetIncome_{t+8}$					2.25	2.42
					(0.71)	(0.74)
$NetIncome_{t+9}$					-2.49	-2.66
					(-1.26)	(-1.26)
$Investment_{t+1}$					-13.66	-9.68
					(-2.14)	(-2.40)
$Investment_{t+2}$					11.85	13.07
					(1.15)	(1.24)
$Investment_{t+3}$					10.55	9.75
					(2.37)	(1.60)
$Investment_{t+4}$					27.30	30.02
					(8.80)	(7.47)
I_t/K_t						-22.18
						(-4.69)
R^2	0.015	0.005	0.116	0.133	0.180	0.181

Table 3.2: Continued.

	(1)	(2)	(3)	(4)	(5)	(6)
Panel B Manufacturing Firms ($1000 \leq SIC < 4000$)						
$Log(Size_t)$	-0.69			-0.65	-0.96	-0.94
	(-3.61)			(-3.55)	(-5.88)	(-5.85)
MB_{t-3}			-0.12	-0.12	-0.14	-0.12
			(-1.61)	(-1.56)	(-2.79)	(-2.37)
MB_{t-2}			-0.37	-0.33	-0.29	-0.29
			(-2.90)	(-2.77)	(-2.77)	(-2.77)
MB_{t-1}			-0.39	-0.37	-0.37	-0.37
			(-3.73)	(-3.72)	(-3.79)	(-3.78)
MB_t		-0.10	-2.69	-2.62	-2.61	-2.60
		(-1.74)	(-7.36)	(-7.29)	(-7.14)	(-7.21)
MB_{t+1}			3.37	3.35	3.46	3.44
			(8.40)	(8.43)	(7.24)	(7.44)
MB_{t+2}			0.54	0.56	0.47	0.47
			(4.23)	(4.30)	(3.08)	(3.17)
MB_{t+3}			0.23	0.23	0.25	0.26
			(2.11)	(1.87)	(2.16)	(2.22)
MB_{t+4}			0.03	0.08	0.14	0.11
			(0.37)	(1.09)	(1.79)	(1.44)
MB_{t+5}			0.03	0.03	-0.03	-0.00
			(0.51)	(0.53)	(-0.34)	(-0.12)
$Inventory_t$				1.46	1.59	1.63
				(4.47)	(3.83)	(4.88)
$NetIncome_{t+1}$					17.58	17.70
					(6.52)	(6.59)
$NetIncome_{t+2}$					34.87	34.99
					(3.69)	(3.66)
$NetIncome_{t+3}$					11.55	11.58
					(2.64)	(2.72)
$NetIncome_{t+4}$					12.61	12.51
					(4.82)	(4.77)
$NetIncome_{t+5}$					7.94	7.90
					(3.85)	(3.85)
$NetIncome_{t+6}$					3.28	2.73
					(1.60)	(1.24)
$NetIncome_{t+7}$					10.78	9.89
					(0.94)	(0.94)
$NetIncome_{t+8}$					0.02	-0.32
					(0.01)	(-0.13)
$NetIncome_{t+9}$					-1.26	-0.62
					(-0.48)	(-0.27)
$Investment_{t+1}$					-37.71	-36.61
					(-1.68)	(-1.59)
$Investment_{t+2}$					40.11	32.73
					(1.18)	(1.31)
$Investment_{t+3}$					14.47	19.16
					(1.39)	(2.35)
$Investment_{t+4}$					39.31	41.20
					(8.48)	(8.97)
I_t/K_t						-16.68
						(-3.10)
R^2	0.015	0.007	0.135	0.154	0.214	0.216

only in Panel B. Several interesting observations arise. First, the average R^2 shows that the model provides a better fit for manufacturing firms than that of the full sample in Panel A. Second, the time delay in production appears to be longer than the full sample case in Panel A. For example, Model (5) estimates $d = 3 \sim 4$ and $h + d = 4$. Overall, the estimation in Panels A and B suggests that $d = 2 \sim 4$ and $h = 1 \sim 2$. Therefore, in the main model of calibrations in the next section, I set $d = 3$ and $h = 2$. The estimated TTB here is shorter than that in Kydland and Prescott (1982), where they assume $h = 3$.

3.3.2 Industry Level Delays

To see the production delays in details, I further examine the distribution of production delays among industries. I run full sample time-series regression of (3.1) for each industry, based on 2-digit SIC codes.[5] Firms with the same 2-digit SIC code are aggregated into one industry. The industry returns are value-weighted average of individual stock returns. To get a more precise distribution of production delays, I set the maximum $d = 5$ and $h = 4$. The sample period is 3/1984-12/2009.

Table 3.3 summarizes the distribution of TTB and TTP among industries. Overall, the estimates appear to be reasonable. For example, industries such as mining, construction, manufacturing, and transportation have longer time-to-build. Also, industries like manufacturing, agriculture, mining and services have longer time-to-produce.

[5] I drop the investment to capital ratio in the regression here to save the degree of freedom. This has no qualitative effects on the regression results, as shown in Table 3.2.

Table 3.3: Estimating Time-to-Build and Time-to-Produce: Industry Level.

This table summarizes the distribution of time-to-build (h) and time-to-produce (d) among industries, estimating from the following full sample time-series regression:

$$\begin{aligned} R_{t,t+1} &= \alpha + \sum_{i=1}^{d-1} \beta_{-i} MB_{t-i} + \beta_0 MB_t + \sum_{i=1}^{d+1} \beta_i MB_{t+i} + \beta_{size} Log(Size_t) + \beta_Q Inventory_t \\ &\quad + \sum_{i=1}^{h} \beta_{I,i} Investment_{t+i} + \sum_{i=1}^{h+d+1} \beta_{NI,i} NetIncome_{t+i} + \varepsilon_{t+1} \end{aligned}$$

where $R_{t,t+1}$ is the quarterly industry return from time t to $t+1$, MB is the ratio of market equity to book equity, computed as in Fama and French (1993), $Size_t$ is the market capitalization, $Inventory_t$ is from Compustat Item INVTQ normalized by $Size_t$, $Investment$ is computed from Compustat Item CAPXY normalized by $Size_t$, and $NetIncome$ is the net income available to common equities (Item IBCOMQ) normalized by $Size_t$. Firms with the same 2-digit SIC codes are aggregated into one industry. Industry returns are the value-weighted average of individual stock returns. Only common stocks (share codes 10 or 11) in the NYSE/AMEX/NASDAQ are included, excluding all financial (SIC code between 6000 and 6999) and utility firms (SIC code between 4900 and 4999) and firms with fiscal quarters ending on a month other than March/June/September/December. The sample period is 3/1984-12/2009.

Panel A		Time-to-build (h)
h	SIC	Industry description
≥ 4	14	Mining And Quarrying Of Nonmetallic Minerals, Except Fuels
3	31	Leather And Leather Products
3	45	Transportation By Air
2	52	Building Materials, Hardware, Garden Supply, And Mobile Home Dealers
2	70	Hotels, Rooming Houses, Camps, And Other Lodging Places
1	13	Oil And Gas Extraction
1	15	Building Construction General Contractors And Operative Builders
1	35	Industrial And Commercial Machinery And Computer Equipment
1	41	Local And Suburban Transit And Interurban Highway Passenger Transportation
1	79	Amusement And Recreation Services
Panel B		Time-to-produce (d)
d	SIC	Industry description
≥ 5	23	Apparel And Other Finished Products Made From Fabrics And Similar Materials
≥ 5	29	Petroleum Refining And Related Industries
≥ 5	42	Motor Freight Transportation And Warehousing
≥ 5	87	Engineering, Accounting, Research, Management, And Related Services
4	39	Miscellaneous Manufacturing Industries
4	80	Health Services
3	1	Agricultural Production Crops
3	22	Textile Mill Products
3	27	Printing, Publishing, And Allied Industries
3	36	Electronic And Other Electrical Equipment And Components, Except Computer Equipment
3	46	Pipelines, Except Natural Gas
3	48	Communications
3	55	Automotive Dealers And Gasoline Service Stations
2	10	Metal Mining
2	33	Primary Metal Industries
2	40	Railroad Transportation
2	45	Transportation By Air
2	52	Building Materials, Hardware, Garden Supply, And Mobile Home Dealers
2	72	Personal Services
2	83	Social Services

4. THE NUMERICAL SOLUTION

Three sources contribute to the computational complexity of this model: (a) the number of state variables; (b) the delays in production, which make the equilibrium conditions more deeply recursive (see Equation (2.24)); (c) the recursive preferences, which complicate computing the pricing kernel. I apply a perturbation method (Judd and Guu (1993, 1997) and Schmitt-Grohé and Uribe (2004)) to solve this DSGE model. A perturbation method is preferred here, instead of a projection method or a value function iteration, for several reasons. First, given the number of state variables and the deeply recursive equilibrium conditions in this model, a perturbation method is the only one that is both computationally feasible and efficient. Second, although it is a local approximation, the perturbation method has proven to be highly accurate in many applications.[1] Third, the perturbation method intuitively demonstrates how risk aversion impacts the economic system, which is useful from an asset pricing perspective.

I implement a third-order perturbation of the model. It is well known that a first-order perturbation is essentially a certainty equivalent, which is similar to the usual linearization approach and not interesting for asset pricing, because all policy functions are independent of risk aversion. A second-order approximation does incorporate risk aversion. However, the risk premium is a constant in this case. In addition, as shown later in Appendix C, a second-order perturbation provides counterfactual function approximations for this model. For example, the value function monotonically decreases with the capital stock. Although it is not quantitatively large, this implies approximation errors in the second-order perturbation. A third-

[1]See Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), Swanson, Anderson, and Levin (2006), Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2009), Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez, and Uribe (2009), and Fernández-Villaverde, van Binsbergen, Koijen, and Rubio-Ramírez (2010).

order approximation uses additional terms relative to the second order approximation and can generate a time-varying risk premium, which is crucial for asset pricing, especially to price the time-varying risks in the model. A higher order approximation, although theoretically appealing, is computationally infeasible in this model. In fact, Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006) find that the derivatives after the 5th-order are numerically insignificant in a neoclassical growth model. Therefore, I use a third-order perturbation.

A perturbation method approximates a function around its steady state (i.e., $\sigma = 0$), using a Taylor expansion. Here I illustrate an approximation of the utility function. For simplicity, assume x_t is only one state variable, then the third order Taylor expansion of utility at time t, $U(x_t, \sigma)$, around the deterministic steady state $(x, 0)$ is

$$\begin{aligned} U\left[x_t, \sigma\right] \;\approx\; & U_{ss} + U_{ss}^{(0,1)}\sigma + \frac{1}{2}U_{ss}^{(0,2)}\sigma^2 + \frac{1}{6}U_{ss}^{(0,3)}\sigma^3 + U_{ss}^{(1,0)}\left(x_t - x\right) \\ & + U_{ss}^{(1,1)}\left(x_t - x\right)\sigma + \frac{1}{2}U_{ss}^{(1,2)}\left(x_t - x\right)\sigma^2 + \frac{1}{2}U_{ss}^{(2,0)}\left(x_t - x\right)^2 \\ & + \frac{1}{2}U_{ss}^{(2,1)}\left(x_t - x\right)^2\sigma + \frac{1}{6}U^{(3,0)}\left(x_t - x\right)^3 \end{aligned} \tag{4.1}$$

where ss indicates the deterministic steady state. The main task here is to compute the partial derivatives at the steady state, which can be obtained by taking partial derivatives of the equilibrium conditions and evaluating at the steady state.

The equilibrium conditions of this model can be described as

$$\mathbb{E}_t\left[\mathbf{F}\left(\mathbf{Y}_t, \mathbf{X}_t, ..., \mathbf{Y}_{t+d+1}, \mathbf{X}_{t+d+1}, \sigma\right)\right] = 0 \tag{4.2}$$

where $\mathbf{X}$ and $\mathbf{Y}$ are the state variables and other variables. Obviously, the partial derivatives of the above equilibrium conditions with respect to the state variables or perturbation parameter are 0 as well. One important observation here is that since σ is always multiplied by the error term ε in the evolution of the productivity shock, every derivative with respect to σ contains the error term as a product. Therefore, when the expectation operator is applied to the equilibrium conditions and evaluated at the steady state, we know that all derivatives with odd powers of σ are 0 since the error terms are IID standard normally distributed. For example, $U_{ss}^{(0,1)} = 0$, $U_{ss}^{(0,3)} = 0$, $U_{ss}^{(1,1)} = 0$, $U_{ss}^{(2,1)} = 0$. Only derivatives with even powers of σ are left. Thus, risk aversion only shows up in the derivatives with even powers of σ where the uncertainty shows up. Therefore, the first order perturbation is a certainty equivalent, in which risk aversion does not matter.

The above approximation scheme applies to all unknown variables in the equilibrium conditions. One advantage of the perturbation method is that we can easily add variables of interest to the equilibrium system. For instance, we can add the risk-free rate equation, $\mathbb{E}[M_{t,t+}R_{f,t+1}] = 1$. Extra care should be taken when handling the recursive preferences. To make it consistent with the general description in (4.2), we need to introduce one auxiliary variable to capture the expected future utility, i.e., $eu_t = \mathbb{E}_t U_{t+1}^{1-\gamma}$.

Summarizing, the equilibrium conditions are:

$$\begin{aligned}
\mu_t =& (1-\beta)\upsilon U_t^{\frac{1}{\psi}} C_t^{-\frac{1}{\psi}} \left[\upsilon + (1-\upsilon)\left(\frac{Q_t}{C_t}\right)^{1-\frac{1}{\phi}}\right]^{\frac{\frac{1}{\phi}-\frac{1}{\psi}}{1-\frac{1}{\phi}}}, \\
\mathbb{E}_t[M_{t,t+1}] =& -\frac{1-\upsilon}{\upsilon}\left(\frac{C_t}{Q_t}\right)^{\frac{1}{\phi}} + \frac{\partial h_t}{\partial Q_t} + 1, \\
q_t =& \mathbb{E}_t\left[M_{t,t+1}\, q_{t+1}\right] - \delta \mathbb{E}_t\left[\sum_{i=1}^{d+1} M_{t,t+i}\, u_{i-1}\, q_{t+i}\right] \\
&+ \mathbb{E}_t\left[M_{t,t+d+1}\left(\alpha K_{t+1}^{\alpha-1} Z_{t+d+1}^{1-\alpha} + q_{t+d+1}\frac{\partial g_{t+d+1}}{\partial K_{t+1}} - \frac{\partial h_{t+d+1}}{\partial K_{t+1}}\right)\right], \\
0 =& \mathbb{E}_t\left[M_{t,t+h}\, q_{t+h}\frac{\partial g_{t+h}}{\partial S_t}\right] - \mathbb{E}_t\left[\sum_{i=0}^{h} M_{t,t+i} w_i\right], \\
C_t =& K_{t-d}^{\alpha} Z_t^{1-\alpha} - \sum_{i=0}^{h} w_i S_{t-i} - h_t - (Q_t - Q_{t-1}), \\
I_t =& \sum_{i=0}^{h} w_i S_{t-i},
\end{aligned}$$

$$
\begin{aligned}
eu_t =& \mathbb{E}_t U_{t+1}^{1-\gamma}, \\
U_t =& \left\{(1-\beta)\left[\upsilon C_t^{\omega} + (1-\upsilon) Q_t^{\omega}\right]^{\frac{1-\gamma}{\omega\theta}} + \beta[\mathbb{E}_t U_{t+1}^{1-\gamma}]^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}}, \\
1 =& \mathbb{E}[M_{t,t+} R_{f,t+1}], \\
P_t =& \mathbb{E}_t[M_{t,t+1}(P_{t+1} + C_{t+1})].
\end{aligned}
$$

The evolution of capital stock follows

$$
K_{t+1} = K_t + g_t - \delta \sum_{i=0}^{d} K_{t-i} u_i.
$$

The pricing kernel is

$$
M_{t,t+1} = \beta \left[\frac{C_{t+1}}{C_t}\right]^{-\frac{1}{\psi}} \left[\frac{\upsilon + (1-\upsilon)\left(\frac{Q_{t+1}}{C_{t+1}}\right)^{1-\frac{1}{\phi}}}{\upsilon + (1-\upsilon)\left(\frac{Q_t}{C_t}\right)^{1-\frac{1}{\phi}}}\right]^{\frac{\frac{1}{\phi}-\frac{1}{\psi}}{1-\frac{1}{\phi}}} \left[\frac{U_{t+1}^{1-\gamma}}{\mathbb{E}_t U_{t+1}^{1-\gamma}}\right]^{1-\frac{1}{\theta}}.
$$

The equilibrium conditions consist of 10 variables (μ, U, C, Q, S, I, q, eu, P, R_f), 8 state variables (K_{t-3}, K_{t-2}, K_{t-1}, K_t, S_{t-2}, S_{t-1}, Q_{t-1}, z_t), and a perturbation parameter (σ). I perturb the equilibrium conditions in levels of the variables, except the stock price, which is perturbed in log given its relatively large magnitude.[2] See Appendix A for additional details, which closely follows Schmitt-Grohé and Uribe (2004) and Caldara, Fernández-Villaverde, Rubio-Ramírez, and Yao (2009). Appendix A also addresses some computational issues in this model.

[2]I also performed perturbations in logs of all variables, and the results are similar.

5. CALIBRATION APPROACH

5.1 Empirical Data

The model is calibrated to the modern sample over 1964-2009, which is more difficult to match.[1] I obtain the annual market return and the risk-free rate data from the annual Fama-French factors. Other macroeconomic data are mainly collected from the NIPA tables. See Appendix B for details. The key moments of macroeconomic quantities and asset returns are reported in the table on page 39. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter.

5.2 Parameters

The parameters chosen are close to the literature, and summarized in Table 5.1. The capital share (α) is 0.358, which is similar to Boldrin, Christiano, and Fisher (2001) and Kaltenbrunner and Lochstoer (2010). The quarterly depreciation rate (δ) is 0.027, which is the average investment/capital ratio over 1964-2009 from the NIPA tables. Taken from Boldrin, Christiano, and Fisher (2001), the persistence of the technology shock (ρ) is 0.95, and the volatility of the technology shock (σ) is 0.021. Christiano and Todd (1996) argue that most of investment expenditure occurs in the later periods, so I set the proportions of a project invested as $\{w_0, w_1, w_2\} = \{0.1, 0.1, 0.8\}$ in the main model,[2] which features a 3-

---

[1] A longer historical dataset shows more volatile consumption growth rate and lower Sharpe ratio (See Campbell and Cochrane (1999)).

[2] This is largely in line with the literature. For example, Christiano and Vigfusson (2003) estimate the investment weights as 0.01, 0.28, 0.48, and 0.23. Christiano and Todd (1996) consider the time-to-plan weights as 0.01, 0.33, 0.33 and 0.33, while Koeva (2001) documents the investment is about 10% in the first year and 90% in the second year. Zhou (2000) also finds lower investment in the initial period. Kuehn (2009) assumes 20% investment expenditure in the initial period and 80% later.

quarter TTB ($h = 2$) and a 4-quarter TTP ($d = 3$). For the benchmark model with a 6-quarter TTB, I set the investment weights as $\{w_0, w_1, w_2, w_3, w_4, w_5\} = \{0.1, 0.1, 0.2, 0.2, 0.2, 0.2\}$. The proportions of capital stock depreciation are set as $\{u_0, u_1, u_2, u_3\} = \{0.25, 0.25, 0.25, 0.25\}$. There is no empirical guidance in choosing the elasticity of substitution between inventory and consumption, ϕ. It appears to be reasonable to assume that ϕ is close to 1, since inventory refers to the finished good in this model. So, I set ϕ to 1 in the main model. I also perturb ϕ to 0.5 or 1.25 as robustness checks.

The time discount (β), the curvature of capital adjustment costs (χ), the EIS (ψ), and the relative risk aversion (γ) are calibrated to match the asset prices. I set $\beta = 0.986$.[3] The curvature of capital adjustment costs (χ) is set to 2, which is in the range examined in the literature.[4] The EIS (ψ) is chosen as 0.03, which is similar to Yogo (2004, 2006) and Gomes, Kogan, and Yogo (2009). Relative risk aversion (γ) is set to 7.5, which is similar to Bansal and Yaron (2004).

Three parameters remain, and all of these are related to inventory: the elasticity of consumption (ν), the inventory cost coefficient (η), and the curvature of inventory holding costs (τ). Since these parameters are new and there is no existing literature as guidance, based on the steady state equations, I choose these three parameters to match the means of the inventory/capital ratio, the output/capital ratio, and the consumption/capital ratio computed from the NIPA tables over 1964-2009. These ratios are 7.38%, 37.23%, and 24.29%, respectively. This gives $\tau = 0.5$, $\eta = 0.04$, $\nu = 0.955$. With $\tau = 0.5$, inventory holding costs are concave in inventory, so the

---

[3]Note Cooley and Prescott (1995) set $\beta = 0.987$.

[4]For example, Jermann (1998), and Boldrin, Christiano, and Fisher (2001) choose 0.23, Beeler (2009) sets χ of $1 \sim 15$, Kaltenbrunner and Lochstoer (2010) use χ of $0.7 \sim 18$.

Table 5.1: Parameters.

This table summarizes parameters used in the calibration. The time unit is a quarter.

Parameters	**Description**	**Value**
Fixed parameters		
α	Elasticity of capital	0.358
δ	Depreciation rate of capital	0.027
υ	Elasticity of consumption	0.955
ρ	Persistence of the technology shock	0.95
σ	Volatility of the technology shock	0.021
η	Inventory cost coefficient	0.04
τ	Curvature of the inventory holding costs	0.5
$\{w_0, w_1, w_2\}$	Proportion invested of a project ($h = 2$)	{0.1, 0.1, 0.8}
$\{w_0, w_1, w_2, w_3, w_4, w_5\}$	Proportion invested of a project ($h = 5$)	{0.1, 0.1, 0.2, 0.2, 0.2, 0.2}
$\{u_0, u_1, u_2, u_3\}$	Proportion depreciated of the capital ($d = 3$)	{0.25, 0.25, 0.25, 0.25}
ϕ	Elasticity of substitution between inventory and consumption	1
Calibrated parameters		
β	Time discount	0.986
γ	Relative risk aversion	7.5
ψ	Elasticity of intertemporal subsititution	0.03
χ	Curvature of the capital adjustment costs	2

marginal cost of inventory is decreasing, which is consistent with the observation that there is a scaling effect of inventory.[5]

5.3 Calibration

The main model is the standard RBC model with capital adjustment costs, inventory, a 3-quarter ($h = 2$) TTB, and a 4-quarter ($d = 3$) TTP. To investigate the roles of inventory, TTB, and TTP, I also present simulation results for 5 other benchmark models in addition to the main model. These are the standard RBC model with capital adjustment costs (Benchmark 1), the standard RBC model with capital adjustment costs and inventory (Benchmark 2), the standard RBC model with capital adjustment costs, inventory, and a 3-quarter ($h = 2$) TTB (Benchmark 3), the standard RBC model with capital adjustment costs, inventory, and a 6-quarter ($h = 5$) TTB (Benchmark 4), and the standard RBC model with capital adjustment costs, inventory, and a 4-quarter ($d = 3$) TTP (Benchmark 5).

Similar to Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), I simulate each model for 1000 paths. Starting from the non-stochastic steady state, each path has 300 periods, and the first 100 periods are a burn-in to eliminate the transition from the deterministic steady state to the ergodic distribution. One unit of time represents a quarter, so each path is 50 years long, which is roughly identical to the length of the empirical sample.

[5]This could be viewed as a local, equilibrium result instead of a global property. For example, after a warehouse has been built, the total per unit cost associated with inventory holding could be decreasing.

5.4 Results

Before discussing the main results, I first inspect the numerical quality of the simulations. Overall, a third-order perturbation performs well. For example, it only introduces small Euler equation errors. As a comparison, a second-order perturbation gives counterfactual function approximations, e.g., the utility decreases with capital stock, while a first-order perturbation produces large mean asset returns. See Appendix C for details.

5.5 Main Results

Table 5.2 reports simulation results of the main model and 5 other benchmark models. The simulation results demonstrate the ability of the main model to match both macroeconomic quantities and asset prices reasonably well. As in the standard RBC model, it is not surprising to see that the model can reasonably match the volatilities of quantities, like output and consumption. Only the volatility of investment is a little lower than the empirical data. This is due to the capital adjustment costs introduced in the model, as observed in the literature (e.g., Boldrin, Christiano, and Fisher (2001), Kuehn (2009), and Guvenen (2009)). TTB also slows down the response of capital stock to the productivity shock. In terms of the new variable introduced in this model, the inventory ratios are also in line with the data, except that the volatility of inventory/consumption ratio is lower than the data. The moments of asset prices are close to the data as well. The mean and volatility of equity returns are 7.43% and 21.36%, while they are 7.31% and 18.50% in the data, respectively. The main model is able to generate volatile equity returns, because of the delays in the production, without resorting to the leverage effect. Traditional production-based models often face insufficient risks in the economy, thus have to apply the leverage effect to obtain more volatile equity returns (See, e.g., Kuehn (2007), Barro (2009), Gourio (2009), and Bhamra, Kuehn, and Strebulaev (2010)).

Table 5.2: Calibrations: Different Models.

This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of 5 benchmark models and the main model, using a third-order perturbation. Benchmark models are the standard RBC model with capital adjustment costs (Benchmark 1), the standard RBC model with capital adjustment costs and inventory (Benchmark 2), the standard RBC model with capital adjustment costs, inventory and a 3-quarter ($h = 2$) time-to-build constraint (Benchmark 3), the standard RBC model with capital adjustment costs, inventory and a 6-quarter ($h = 5$) time-to-build constraint (Benchmark 4), and the standard RBC model with capital adjustment costs, inventory and a 4-quarter ($d = 3$) time-to-produce constraint (Benchmark 5). The main model is the standard RBC model with capital adjustment costs, inventory, a 3-quarter ($h = 2$) time-to-build constraint, and a 4-quarter ($d = 3$) time-to-produce constraint. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964-2009. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio and the pricing kernel.

	U.S. Data (1964-2009)	Benchmark (1) No Inventory (h=0, d=0)	Benchmark (2) Inventory (h=0, d=0)	Benchmark (3) Inventory (h=2, d=0)	Benchmark (4) Inventory (h=5, d=0)	Benchmark (5) Inventory (h=0, d=3)	Main Model Inventory (h=2, d=3)
Panel A Macroeconomic quantities							
Volatility of output							
$\sigma(Y)$	1.55	1.72	1.72	1.67	1.63	1.56	1.54
Volatility of consumption							
$\sigma(C)$	0.83	0.62	0.60	0.56	0.56	0.79	0.63
Volatility of investment							
$\sigma(I)$	5.28	5.15	5.46	6.96	5.54	3.04	3.75
Mean and volatility of the inventory/consumption ratio							
Q/C	30.68	N/A	11.43	12.24	13.11	28.77	29.91
$\sigma(Q/C)$	7.92	N/A	0.68	2.19	2.83	1.08	3.16
Mean and volatility of the inventory/capital ratio							
Q/K	7.38	N/A	1.03	1.10	1.23	6.68	7.00
$\sigma(Q/K)$	1.69	N/A	0.13	0.24	0.31	0.99	1.29
Panel B Asset prices							
Mean and volatility of the equity returns							
$E[R]$	7.31	5.97	6.00	8.10	8.99	6.93	7.43
$\sigma(R)$	18.50	8.46	8.84	22.44	26.54	18.83	21.36
Mean and volatility of the risk-free rate							
$E[R_f]$	1.73	4.79	4.74	2.68	1.57	2.51	1.86
$\sigma(R_f)$	2.05	0.68	0.73	7.97	9.02	2.06	2.39
Equity premium							
$E[R - R_f]$	5.58	1.17	1.26	5.42	7.42	4.42	5.57
Sharpe ratio							
$E[R - R_f]/\sigma(R)$	0.30	0.14	0.14	0.24	0.28	0.23	0.26
Pricing kernel M							
$\sigma(M)/E(M)$	N/A	0.21	0.22	0.27	0.33	0.35	0.36

More importantly, the main model can generate a risk-free rate with a low volatility of 2.38%. Excessively volatile risk-free rates have been a challenge for the production-based models (Jermann (1998), Boldrin, Christiano, and Fisher (2001), Kuehn (2009), Campanale, Castro, and Clementi (2010), and Kaltenbrunner and Lochstoer (2010)), since the volatility of the risk-free rate moves with that of equity returns. That is part of the tight connection between the equity premium puzzle and the risk-free rate puzzle. However, in the main model here, inventory is a quasi-risk-free asset in the economy. The risk-free rate is defined as the difference between the marginal rate of substitution between consumption and inventory, and the marginal inventory holding cost. Moreover, these two components are positively correlated, i.e., both are counter-cyclical. So, the risk-free rate can be less volatile in the model. Finally, the pricing kernel in the economy is highly volatile and satisfies the Hansen-Jagannathan bound. The ratio of expected and standard deviation of the pricing kernel is 0.36, which is higher than the Sharpe ratio (0.3) observed in the data.

Table 5.3 further presents the autocorrelations and cross-correlations of macroeconomic quantities. The main model displays less persistence properties than the empirical data, i.e., the autocorrelations tend to die out faster than those in the data. The cross-correlations reveal two main discrepancies between the main model and the data. First, investment moves with output simultaneously in the data, however, it lags output by 2 quarters in the model because of TTB. This might attribute to the aggregation process of investment in the data, since it summarizes different stages of investments across different firms. Second, inventory tends to lead output by 2 quarters in the data while no such pattern presents in the main model. The reason might be that inventory in the data includes intermediate goods, which is not considered in the model.

Now, I study the contribution of different features to the model, namely, inventory, TTB, and TTP. In Table 5.2, all 6 models produce similar results for the macroeconomic quantities, except the moments of inventory. Benchmark (2) tells us

Table 5.3: Autocorrelations and Cross-Correlations.

This table summarizes the autocorrelations of output, consumption, investment, and inventory, and the cross-correlations of consumption, investment, and inventory with output. The empirical data are from the NIPA tables over 1964-2009. For the main model, correlations are computed in each sample path and the average over the 1000 sample paths is reported.

Panel A		Autocorrelation $AR(k)$								
	k	1	2	3	4					
Output (Y)	Data	0.86	0.67	0.45	0.23					
	Main Model	0.62	0.33	0.11	-0.03					
Consumption (C)	Data	0.87	0.70	0.52	0.31					
	Main Model	0.62	0.52	0.38	0.22					
Investment (I)	Data	0.90	0.72	0.50	0.28					
	Main Model	0.60	0.30	0.07	0.00					
Inventory (Q)	Data	0.88	0.69	0.48	0.28					
	Main Model	0.71	0.28	0.03	-0.11					
Panel B		Cross-Correlation with Output ($Corr(X_t, Y_{t-k})$)								
	k	-4	-3	-2	-1	0	1	2	3	4
Consumption (C)	Data	0.17	0.37	0.55	0.71	0.83	0.83	0.73	0.57	0.38
	Main Model	-0.04	0.08	0.27	0.53	0.88	0.55	0.52	0.41	0.28
Investment (I)	Data	0.21	0.45	0.66	0.82	0.90	0.82	0.65	0.45	0.27
	Main Model	-0.05	-0.05	0.06	0.25	0.51	0.73	0.94	0.42	0.12
Inventory (Q)	Data	0.65	0.76	0.80	0.74	0.58	0.34	0.16	0.02	-0.09
	Main Model	0.06	0.16	0.33	0.56	0.82	0.65	0.11	-0.18	-0.32

that it is unable to match the moments of inventory if we simply add inventory to the standard RBC model, although it does smooth consumption slightly. The inventory demand in Benchmark (2) is too small relative to the data. For example, the ratio of inventory to capital is only 1.03%, compared to 7.38% in the data. Moreover, as shown in Benchmarks (3) and (4), incorporating TTB contributes little to matching the inventory data as well. Benchmark (5) shows that TTP is necessary to match the moments of inventory. The reason is that when facing a productivity shock, the agent can adjust output via investment quickly if there is no TTP constraint, since the productivity of current capital stock is observable. Hence, inventory is less important in this case. However, with the TTP constraint, since the productivity of current capital stock is unobservable, she has to heavily use inventory to smooth consumption. Thus, inventory is more substantial in the case of TTP.

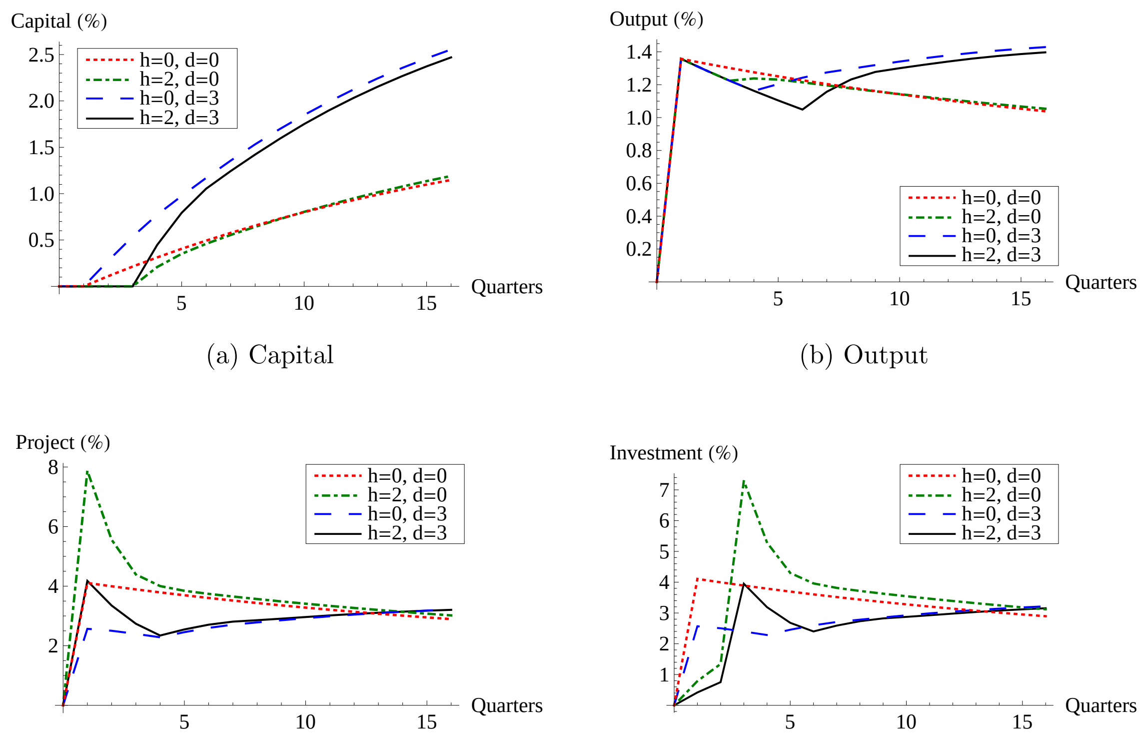
The asset pricing moments provide additional information on the role of inventory, TTB, and TTP. First, Benchmarks (1) and (2) indicate that inventory alone does not help explain the equity premium. All asset price moments in Benchmarks (1) and (2) are quite similar with volatilities that are too low and a risk-free rate that is too high. Second, TTB alone is able to generate a sizable equity premium, but this comes at the cost of producing extremely volatile asset prices, including the risk-free rate. The Sharpe ratio monotonically increases from Benchmarks (2) to (4) as the length of TTB increases. This is accompanied by a monotone increase in the equity return and a decrease in the risk-free rate. For example, the Sharpe ratio is 0.27 in Benchmark (4), compared to 0.14 in the model without TTB (Benchmark (2)). However, even with only a 3-quarter TTB (Benchmark (3)), the volatilities of equity returns and the risk-free rate are 22.44% and 7.97%, respectively. Both rise even more when the length of TTB increases to 6 quarters in Benchmark (4). Third, TTP can help to lower the volatility of asset prices while generating a sizable equity premium, because inventory holdings increase significantly with TTP. Here inventory helps smooth consumption. This is evident from Benchmark (5) and the main model. For example, Benchmark (5) provides an equity premium of 4.42% with a low volatility of the risk-free rate of 2.06%. If we move from Benchmark (4) to the main model, the equity premium and the Sharpe ratio are similar, but the volatility of the risk-free rate shrinks substantially. Comparing Benchmark (3) with Benchmark (5), again we see that TTP is similar to TTB in terms of generating the equity premium, but only TTP together with inventory can achieve a low volatility of the risk-free rate.

In short, Table 5.2 illustrates that although both TTB and TTP are helpful in generating a sizable equity premium, TTB also produces excessively volatile asset prices. Moreover, without the TTP feature, inventory alone is unable to match both macroeconomic quantities and asset prices since inventory holdings are negligible. Only combining TTP with inventory can help to reduce the excess volatility of asset prices relative to the data, which has been a challenge in traditional production-based models.

5.6 Impulse Responses

To understand the mechanism of the main model, it is instructive to study the impulse responses of the key variables in the model. Figure 5.1 displays the responses of capital (K), output (Y), new projects (S), investment (I), consumption (C), and inventory (Q) after a positive, one-standard-deviation technology shock at time 1, as a percentage deviation from the steady state values. For comparison, the plots include the main model ($h = 2, d = 3$), Benchmark 1 (no inventory, $h = 0, d = 0$), Benchmark 3 ($h = 2, d = 0$), and Benchmark 5 ($h = 0, d = 3$).

One striking feature in Panels (b)-(e) of Figure 5.1 is that the TTB and TTP constraints generate cyclical patterns in macroeconomic quantities, but in different dimensions. Different from the monotone patterns from the standard RBC model ($h = 0, d = 0$) and the TTB only model ($h = 2, d = 0$), models with the TTP feature generate an additional twisting point besides time 1 in output, new projects,



(a) Capital

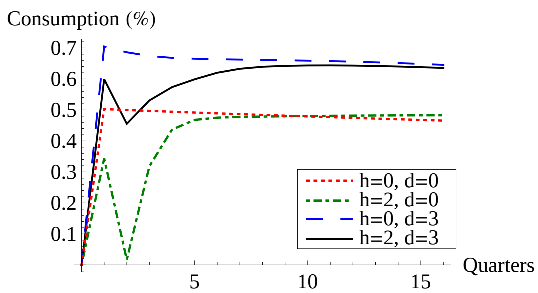
(b) Output

(c) New Project

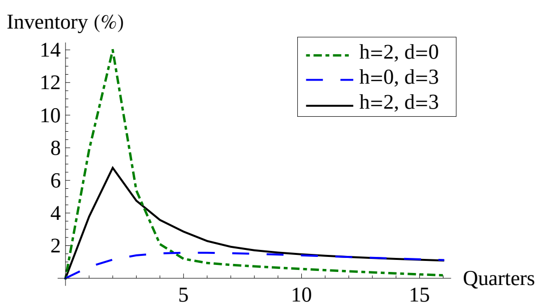
(d) Investment

Fig. 5.1.: Impulse Response Functions.

This figure depicts the impulse response functions of various models after a positive, one-standard-deviation technology shock at time 1. These plots include the standard RBC without inventory ($h = 0, d = 0$), the standard RBC model with inventory and a 3-quarter time-to-build ($h = 2, d = 0$), the standard RBC model with inventory and a 4-quarter time-to-produce ($h = 0, d = 3$), and the standard RBC model with inventory, a 3-quarter time-to-build constraint, and a 4-quarter time-to-produce constraint ($h = 2, d = 3$). Each variable is plotted as a percentage deviation from its steady state value.



(e) Consumption



(f) Inventory

Fig. 5.1.: Continued.

investment, and inventory. Moreover, compared to the standard RBC model ($h = 0, d = 0$) and the TTP only model ($h = 0, d = 3$), models with TTB generate an additional twisting point besides time 1 in consumption. In models with TTB constraint, the decrease in consumption at time 2 results from decreasing output and increasing investments due to the TTB constraint. Moreover, as shown in Panels (b)-(e), the cyclicality of the main model only exists in the initial periods. This implies that the delays in production are, indeed, short-run risks. That explains why a small elasticity of intertemporal substitution is necessary to positively price these risks. Additionally, the productivity shock has different long-term impacts on macroeconomic quantities for models with and without TTP. While there is no difference in investments, models with TTP have larger increases in the capital stock, output, consumption, and inventories after the technology shock.

Next, I turn to specific details of the impulse response functions of the main model. Although the size of a new project increases sharply after observing a positive technology shock, the capital stock stays constant until time 4 because of a 3-quarter TTB. After that, the capital stock climbs steadily. Since output over the first 6 periods depends on the capital stocks up to time 3 (because of a 4-quarter TTP), output decreases in the first 6 periods after a spike at time 1. This fall after the spike is driven by a decay in the technology shock and the fact that new projects have not become productive yet. After that, output rises as the capital stock increases and dominates the decaying technology shock. So, TTP generates the cyclicality in output. Total investment increases in the first 3 periods as the new projects are added and the large weight of investment expenditure in the later period of these projects, and then drops until time 6. The decrease in total investment results from decreasing output until time 6. In turn, the declines in total investment from time 3 to 6 require drops in the size of the new projects from time 1 to 4 due to the 3-quarter TTB.

Panels (d)-(f) of Figure 5.1 highlight how inventory helps smooth consumption. Both consumption and inventory increase initially after the technology shock. Then consumption declines at time 2 since output decreases and both inventory and investment increase. The increase in inventory and the decrease in consumption at time 2 is somewhat surprising as one might expect the agent to invest less in inventory to increase her consumption at time 2. However, since there is a sharp increase in total investment at time 3 while output decreases, the agent wants to save more at time 2 as inventory to compensate for an expected consumption drop at time 3. This exactly captures the consumption smoothing role of inventory. Since the agent prefers to smooth consumption more over time than across states, she uses inventory to smooth consumption. Compared to the benchmark model ($h = 2, d = 0$), the main model generates smoother consumption, because the agent uses more inventory in the presence of the TTP constraint.

Summarizing, the impulse responses demonstrate that the delays in production can generate cyclical patterns and the cyclicality only exists in the initial periods. This suggests a small elasticity of intertemporal substitution to positively price the short-run risks. In addition, inventory acts as a buffer to smooth consumption, which is useful in filtering the volatilities of asset returns. Therefore, the impulse responses provide an intuitive way to interpret the calibration results of the main model.

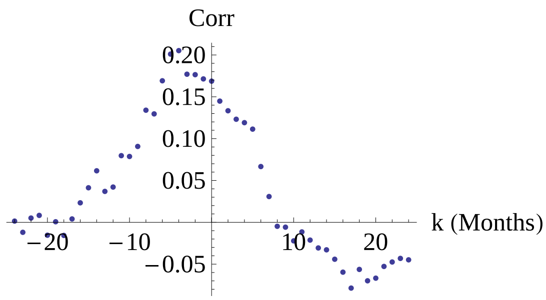
5.7 Asset Prices and the Business Cycle

Empirically, asset prices tend to lead the business cycle. For example, Backus, Routledge, and Zin (2007, 2010) examine the cyclical component of various asset prices, including equity returns, bond yields and commodities. They find robust evidence that financial variables lead macroeconomic quantities by roughly 2 quarters.[6] These cross-correlations are at odds with the standard real business cycle models, where everything moves simultaneously. Now I ask whether delays in production can generate the lead-lag patterns observed in the data, and more importantly, which component contributes to these patterns.

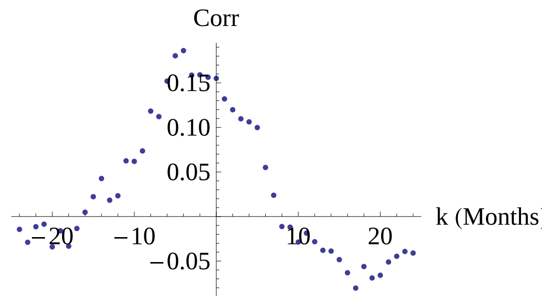
Following Backus, Routledge, and Zin (2010), I compute the cross-correlation between returns and consumption growth as $Corr\,(Return_t, \Delta C_{t-k})$. If the correlations are large when $k > 0$, then consumption growth leads the returns. If the correlations are large when $k < 0$, then returns lead consumption growth. The returns are measured as either market returns or excess market returns. Panels (a) and (b) of Figure 5.2 plot the cross-correlation between market returns or excess market returns and consumption growth over 1964-2009, using monthly data. As in Backus, Routledge, and Zin (2010), the consumption growth rates are computed from year-to-year growth rates, like $\Delta C_t = log(C_{t+6}) - log(C_{t-6})$. The larger and significant correlations appear at $k = -4, -5$ in Figure 5.2(a) , which are 0.21 and 0.19, respectively. So, market returns appear to be a leading indicator of consumption growth. Similar findings can be seen in Figure 5.2(b). In fact, this pattern is robust to different measures of macroeconomic quantities, e.g., industrial production or employment. Untabulated results show that market returns lead employment by roughly 8 months with a correlation of 0.21, and lead industrial production by about 7 months with a correlation of 0.25. In short, asset prices lead macroeconomic quantities by roughly 2 quarters.

Panels (c) and (d) depict cross-correlations between equity returns and consumption growth based on the simulated quarterly data from the main model. The cross-correlation is computed in each sample path first and the average cross-correlation

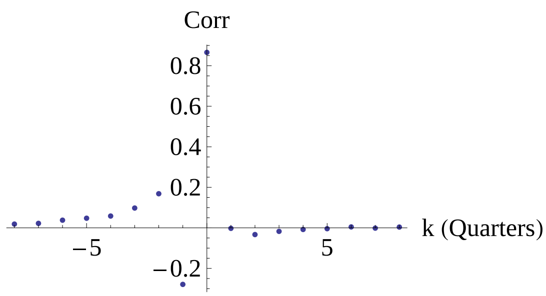
[6]Fama and French (1989), King and Watson (1996), Stock and Watson (2003), Ang, Piazzesi, and Wei (2006), and Beaudry and Portier (2006) also document similar patterns.



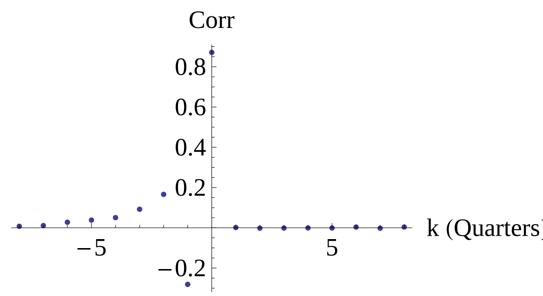
(a) $Corr\,(R_{M,t}, \Delta C_{t-k})$: Data (1964-2009)



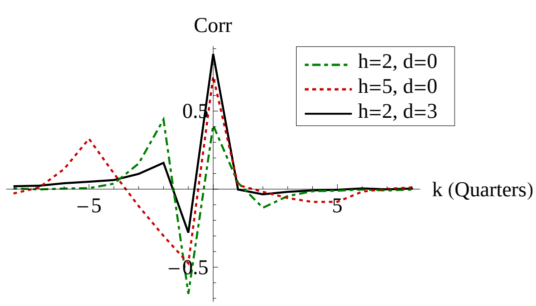
(b) $Corr\,(R_{M,t} - R_{f,t}, \Delta C_{t-k})$: Data (1964-2009)



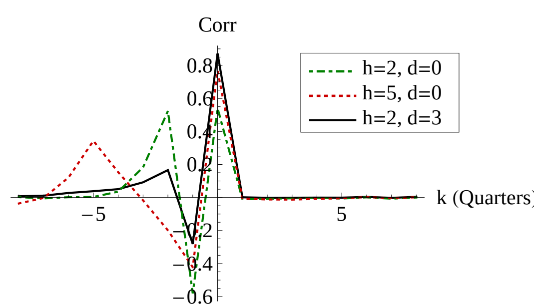
(c) $Corr\,(R_t, \Delta C_{t-k})$: Main Model



(d) $Corr\,(R_t - R_{f,t}, \Delta C_{t-k})$: Main Model



(e) $Corr\,(R_t, \Delta C_{t-k})$: Different Models



(f) $Corr\,(R_t - R_{f,t}, \Delta C_{t-k})$: Different Models

Fig. 5.2.: Cross Correlations between Returns and Consumption Growth Rates.

Figure (a) and (b) plot the cross-correlation between market returns or excess market returns with consumption growth rates over 1964-2009, using monthly data. Figure (c) - (f) depict the cross-correlation from the simulated quarterly data from various models. The cross-correlation is computed in each sample path and the average over the 1000 sample paths is reported.

is reported. Care should be taken when interpreting the correlation at $k = 0$ and $k = -1$. The large positive correlation at $k = 0$ reflects the fact that there is only one shock in the model and the shock is an AR(1) process, so everything moves simultaneously as in the traditional RBC models. The large negative correlation at $k = -1$ is due to a mechanical relation between consumption and equity returns. Since in the model, the dividend equals consumption, when consumption is low at $t - 1$, which corresponds to a low realized return at $t - 1$ given the low dividend, consumption growth will be high at t. Therefore, this introduces a negative correlation between the return at $t - 1$ and consumption growth at t. After discarding the uninformative points at $k = 0$ and $k = -1$, Panels (c) and (d) are close to those of Panels (a) and (b), including magnitudes. For example, the largest correlations between returns and consumption growth are 0.17 when $k = 2$ in Panels (c) and (d). So, the main model is capable of generating the lead-lag correlation patterns observed in the data.

To what extent do TTB and TTP contribute to the lead-lag patterns documented above? Panels (e) and (f) of Figure 5.2 plot cross-correlations computed from the main model ($h = 2, d = 3$), the benchmark model ($h = 2, d = 0$), and the benchmark model ($h = 5, d = 0$). These plots display that the large correlations occur at $k = -5$ for the benchmark model ($h = 5, d = 0$), and at $k = -2$ for the main model and the benchmark model ($h = 2, d = 0$). So, the benchmark model ($h = 5, d = 0$) is inconsistent with the data in terms of the length of the lags. Clearly, the length of TTB determines the length of the lags between returns and consumption growth. If we compare the main model ($h = 2, d = 3$) and the benchmark model ($h = 2, d = 0$), we see that the correlations generated by the benchmark model ($h = 2, d = 0$) are too large compared to the data, which are 0.45 and 0.52 at $k = -2$ in Panels (e) and (f), respectively. Also, these correlations die out quickly in the model with a 3-quarter TTB only ($h = 2, d = 0$).

Summarizing, the length of TTB creates the lags of lead-lag patterns found in the data, while the TTP feature helps to match the cross-correlation quantitatively and produces a persistent cross-correlation. Both features are necessary for the success of the main model to match both the length of lags and the magnitude of correlations.

5.8 Return Predictability

Stock and bond returns tend to move together (e.g., Baele, Bekaert, and Inghelbrecht (2010)). Empirically, the correlation between quarterly stock and bond returns in U.S. markets is 0.17 during 1964-2009. In the main model, since the pricing kernel is counter-cyclical while the equity return is procyclical, the risk-free rate comoves with the equity return. Quantitatively, the average correlation between stock returns and the risk-free rate from the main model is 0.17, which is similar to the empirical data.

Although suffering from measurement and econometric methodology problems, previous studies typically find that the dividend-price ratio can predict future returns.[7] Here, I explore what the main model delivers with respect to return predictability. This provides conditional asset pricing implications of the model, in addition to the unconditional moments reported before.

Table 5.4 reports predictability regressions of the dividend-price ratio. The empirical data are from the NYSE/AMEX/NASDAQ annual market returns over 1925-2009 from CRSP, deflated by the CPI. The regression results are similar to Cochrane (2008a,b). That is, returns can be predicted by the dividend-price ratio while divi-

[7]See LeRoy and Porter (1981), Shiller (1981), Campbell and Shiller (1988), and Fama and French (1988) for early work, and Cochrane (2008a,b) for a recent summary. Stambaugh (1999), Lewellen (2004), Campbell and Yogo (2006), Goyal and Welch (2003), and Lettau and Nieuwerburgh (2008) discuss empirical methodology issues. Robertson and Wright (2006), Boudoukh, Michaely, Richardson, and Roberts (2007), and Larrain and Yogo (2008) investigate different payout measures.

Table 5.4: Return Predictability.

This table presents predictability regressions of the dividend-price ratio. The annual U.S. market returns (including NYSE/AMEX/NASDAQ) from CRSP over 1925-2009, deflated by the CPI, are used. The simulated data from the main model are firstly aggregated into annual, and the median values of regressions over the 1000 sample paths are reported. The t-statistics are corrected for the heteroscedasticity and autocorrelation, using Newey-West standard errors.

Panel A	Regression: $R_{t,t+k} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$					
Horizon	U.S. Data (1925-2009)			Main Model		
k (years)	b	$t(b)$	R^2	b	$t(b)$	R^2
1	3.50	2.39	0.062	3.28	2.32	0.088
2	6.89	2.89	0.107	6.72	2.82	0.166
Panel B	Regression: $\frac{D_{t+k}}{D_t} = a + b\frac{D_t}{P_t} + \varepsilon_{t+k}$					
Horizon	U.S. Data (1925-2009)			Main Model		
k (years)	b	$t(b)$	R^2	b	$t(b)$	R^2
1	0.03	0.02	0.000	-0.01	-0.13	0.006
2	-0.35	-0.19	0.001	0.09	0.77	0.022

dend growth is unpredictable. The return predictability increases with the horizon as these variables are persistent. I run similar regressions over each sample path of the simulated data generated from the main model. The median values of the regressions are reported. The results demonstrate that the main model closely matches the predictability of the dividend-price ratio observed in the data, including its magnitude.[8]

[8]However, this can not be interpreted as evidence to support the view that variation in the dividend-price ratio mainly comes from discount rates, since the model is not designed to address the debate on sources of return predictability.

5.9 Sensitivity Analyses

5.9.1 Exploring the Elasticity of Intertemporal Substitution

From the previous section, a small elasticity of intertemporal substitution is necessary to positively price the short-lived risks introduced by the delays in production. Here, I further explore effects of the EIS on macroeconomic quantities and asset prices. Table 5.5 presents numerical results of alternative models with different EIS values. The table includes two low EIS cases with $\psi = 0.03$ (the main model) and $\psi = 0.06$, respectively, which imply the agent prefers late resolution of uncertainty, the case of the constant relative risk aversion ($\psi = 1/\gamma$), and two cases of high EIS ($\psi = 0.5$ and $\psi = 1.5$), which indicate the agent favors early resolution of uncertainty.

Examining the macroeconomic quantities in Table 5.5, the volatility of output does not vary a lot with the EIS, but the volatility of consumption substantially increases with the EIS while the investment and inventory become less volatile when the EIS rises. The reason is that the propensity of smoothing consumption over time weakens when the EIS increases. Thus, the agent is willing to accept a more volatile consumption stream when her elasticity of intertemporal substitution is high. Given the fixed volatility of aggregate output, we see that the volatilities of investment and inventory decline with EIS. The volatilities of consumption and investment are far away from the data when the EIS is high. For instance, the volatility of consumption is even higher than the volatility of output when $\psi = 1.5$. Turning to the asset prices, the price of risk decreases with the EIS since the agent is less averse to the intertemporal substitution. Consequently, the equity premium and the Sharpe ratio drop significantly when the EIS is high. For example, the equity premium is only 0.17% when $\psi = 1.5$. The overall evidence in Table 5.5 demonstrates that a small EIS is required to match both macroeconomic quantities and asset prices.

Table 5.5: Calibrations: Different Elasticity of Intertemporal Substitution.

This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of alternative models with different elasticity of intertemporal substitution (ψ), using a third-order perturbation. These models feature with inventory, a 3-quarter ($h = 2$) time-to-build constraint, and a 4-quarter ($d = 3$) time-to-produce constraint. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964-2009. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio and the pricing kernel.

	U.S. Data	Prefer Late Resolution of Uncertainty		CRRA	Prefer Early Resolution of Uncertainty	
	(1964-2009)	$\psi = 0.03$	$\psi = 0.06$	$\psi = 1/\gamma$	$\psi = 0.5$	$\psi = 1.5$
Panel A Macroeconomic quantities						
Volatility of output						
$\sigma(Y)$	1.55	1.54	1.56	1.59	1.63	1.66
Volatility of consumption						
$\sigma(C)$	0.83	0.63	0.86	1.14	1.52	1.70
Volatility of investment						
$\sigma(I)$	5.28	3.75	3.28	2.70	1.85	1.43
Mean and volatility of the inventory/consumption ratio						
Q/C	30.68	29.91	29.87	29.70	29.43	29.33
$\sigma(Q/C)$	7.92	3.16	2.59	2.02	1.34	1.07
Mean and volatility of the inventory/capital ratio						
Q/K	7.38	7.00	7.20	7.34	7.40	7.41
$\sigma(Q/K)$	1.69	1.29	1.04	0.76	0.51	0.47
Panel B Asset prices						
Mean and volatility of the equity returns						
$E[R]$	7.31	7.43	6.67	6.09	5.76	5.72
$\sigma(R)$	18.50	21.36	15.76	10.11	4.21	2.04
Mean and volatility of the risk-free rate						
$E[R_f]$	1.73	1.86	3.33	4.48	5.32	5.55
$\sigma(R_f)$	2.05	2.39	1.85	1.26	0.53	0.22
Equity premium						
$E[R - R_f]$	5.58	5.57	3.34	1.61	0.44	0.17
Sharpe ratio						
$E[R - R_f]/\sigma(R)$	0.30	0.26	0.21	0.16	0.10	0.08
Pricing kernel M						
$\sigma(M)/E(M)$	N/A	0.36	0.21	0.15	0.10	0.08

5.9.2 Exploring the Relative Risk Aversion

In this subsection, I examine the sensitivity of the macroeconomic quantities and asset prices to risk aversion. This provides another angle for us to differentiate contributions of risk aversion and intertemporal substitution. Table 5.6 summarizes cases of a low risk aversion ($\gamma = 5$), the main model ($\gamma = 7.5$), and a high risk aversion ($\gamma = 10$). Table 5.6 shows that the effects of relative risk aversion on the macroeconomic quantities are negligible. In contrast, its impact can be seen from the asset prices, in particular, the price of risk. For example, the Sharpe ratio increases from 0.24 to 0.27 when γ shifts from 5 to 10. The pricing kernel becomes more volatile when relative risk aversion rises. Overall, the main model seems to provide a reasonable fit to the data in terms of both macroeconomic quantities and asset prices.

5.9.3 Exploring the Elasticity of Substitution Between Inventory and Consumption

Given the lack of empirical evidence regarding the elasticity of substitution between inventory and consumption, it is worth studying the impact of ϕ on macroeconomic quantities and asset prices. Columns 2-4 of Table 5.7 present numerical results for cases of a relatively small elasticity of substitution (ϕ=0.5), the main model (ϕ=1), and a relatively large elasticity of substitution (ϕ=1.25). Clearly, inventory holdings decrease with the elasticity of substitution between inventory and consumption. For example, the mean inventory/consumption drops from 66.64% to 14.06% when ϕ changes from 0.5 to 1.25. As a result, the risk-free rate becomes more volatile. For instance, the volatility of risk-free rate increases from 2.28% to 3.42%. Even the mean risk-free rate decreases with ϕ since the marginal inventory holding costs increase. But, overall, the asset pricing moments seem to be less sensitive to ϕ. This gives us confidence of the main model.

Table 5.6: Calibrations: Different Relative Risk Aversion.

This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of alternative models with different relative risk aversion (γ), using a third-order perturbation. These models feature with inventory, a 3-quarter ($h = 2$) time-to-build constraint, and a 4-quarter ($d = 3$) time-to-produce constraint. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964-2009. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio and the pricing kernel.

	U.S. Data (1964-2009)	$\gamma = 5$	$\gamma = 7.5$	$\gamma = 10$
Panel A Macroeconomic quantities				
Volatility of output				
$\sigma(Y)$	1.55	1.54	1.54	1.54
Volatility of consumption				
$\sigma(C)$	0.83	0.64	0.63	0.63
Volatility of investment				
$\sigma(I)$	5.28	3.77	3.75	3.74
Mean and volatility of the inventory/consumption ratio				
Q/C	30.68	29.86	29.91	29.97
$\sigma(Q/C)$	7.92	3.15	3.16	3.12
Mean and volatility of the inventory/capital ratio				
Q/K	7.38	7.11	7.00	6.97
$\sigma(Q/K)$	1.69	1.32	1.29	1.27
Panel B Asset prices				
Mean and volatility of the equity returns				
$E[R]$	7.31	7.61	7.43	7.44
$\sigma(R)$	18.50	21.22	21.36	21.44
Mean and volatility of the risk-free rate				
$E[R_f]$	1.73	2.42	1.86	1.68
$\sigma(R_f)$	2.05	2.45	2.39	2.32
Equity premium				
$E[R - R_f]$	5.58	5.19	5.57	5.76
Sharpe ratio				
$E[R - R_f]/\sigma(R)$	0.30	0.24	0.26	0.27
Pricing kernel M				
$\sigma(M)/E(M)$	N/A	0.31	0.36	0.39

Table 5.7: Calibrations: Different Inventory Specifications.

This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of models with inventory-in-the-utility or inventory-in-the-production specifications, using a third-order perturbation. Models with different elasticities of substitution between consumption (capital) and inventory are illustrated. All models feature capital adjustment costs, inventory, a 3-quarter ($h = 2$) time-to-build constraint, and a 4-quarter ($d = 3$) time-to-produce constraint. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964-2009. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio and the pricing kernel.

	U.S. Data	Inventory-in-Utility			Inventory-in-Production		
	(1964-2009)	ϕ=0.5	ϕ=1	ϕ=1.25	ϕ=0.5	ϕ=1	ϕ=1.25
Panel A Macroeconomic quantities							
Volatility of output							
$\sigma(Y)$	1.55	1.54	1.54	1.55	1.62	1.68	1.75
Volatility of consumption							
$\sigma(C)$	0.83	0.71	0.63	0.63	0.82	0.85	0.89
Volatility of investment							
$\sigma(I)$	5.28	3.71	3.75	3.99	3.19	3.56	4.20
Mean and volatility of the inventory/consumption ratio							
Q/C	30.68	66.64	29.91	14.06	141.89	29.70	5.69
$\sigma(Q/C)$	7.92	3.12	3.16	2.88	24.18	4.58	1.78
Mean and volatility of the inventory/capital ratio							
Q/K	7.38	15.75	7.00	3.32	44.72	7.60	1.39
$\sigma(Q/K)$	1.69	2.11	1.29	0.94	2.64	0.97	0.51
Panel B Asset prices							
Mean and volatility of the equity returns							
$E[R]$	7.31	7.40	7.43	7.87	6.90	7.31	8.61
$\sigma(R)$	18.50	21.21	21.36	23.22	18.37	20.90	26.14
Mean and volatility of the risk-free rate							
$E[R_f]$	1.73	1.95	1.86	1.34	0.42	1.77	0.39
$\sigma(R_f)$	2.05	2.28	2.39	3.42	3.33	2.33	5.17
Equity premium							
$E[R - R_f]$	5.58	5.45	5.57	6.53	6.48	5.54	8.22
Sharpe ratio							
$E[R - R_f]/\sigma(R)$	0.30	0.26	0.26	0.28	0.35	0.27	0.31
Pricing kernel M							
$\sigma(M)/E(M)$	N/A	0.36	0.36	0.37	0.35	0.37	0.38

5.10 Alternative Inventory Specification

Different from the reduced form approach used in this paper, the benefit of holding inventory has been mostly modeled through production. For comparisons, I study the inventory-in-the-production specification in this subsection. Similar to Kydland and Prescott (1982), Christiano (1988), Gomes, Kogan, and Yogo (2009), Belo and Lin (2009), and Jones and Tuzel (2010)), inventory serves as a factor input into the production. The production function is

$$y_t = Z_t^{1-\alpha} \left[\upsilon K_{t-d}^{\omega} + (1-\upsilon) Q_{t-1}^{\omega} \right]^{\frac{\alpha}{\omega}}, \tag{5.1}$$

where $\omega = 1 - \frac{1}{\phi}$ and ϕ is the elasticity of substitution between inventory and capital.

Columns 5-7 of Table 5.7 summarize calibration results for models with inventory-in-the-production, which include cases of a relatively small elasticity of substitution (ϕ=0.5), a unitary elasticity of substitution (ϕ=1), and a relatively large elasticity of substitution.[9] Parameter υ is pinned down by setting the steady state inventory/capital ratio as 7.38%, which is the mean inventory/capital ratio over 1964-2009. This gives $\upsilon = 0.919$. The results show that a model with inventory-in-the-production specification also matches macroeconomic quantities and asset prices reasonably well. For example, when $\phi = 1$, the model generates an equity premium of 5.54% per year and a low volatility of risk-free rate of 2.33%. Again, we see the consumption smoothing role of inventory, even though inventory enters as an input factor into production. The reason is that inventory is a much less risky input factor than capital, as shown in (5.1). More importantly, the results are close to a model with inventory-in-the-utility specification. This validates that in a one-firm and one-agent setting, inventory-in-the-utility specification can be viewed as a reduced form of inventory-in-the-production specification. However, asset prices seem to be more

[9]Kydland and Prescott (1982) suggest a small elasticity of substitution of 0.2, while Belo and Lin (2009) set it to 1 and Jones and Tuzel (2010) set it close to 1.

sensitive to the elasticity of substitution for models with inventory-in-the-production specification, especially the risk-free rate. For instance, both mean and volatility of the risk-free rate deviate significantly from the data when ϕ deviates from 1.

6. CONCLUSIONS

This paper studies equilibrium asset prices and macroeconomic quantities in a dynamic production-based equilibrium model with time-to-build (TTB), time-to-produce (TTP), and inventory. These delays in production expand the state space of the economy to include both historical and future capital stocks, in addition to the current capital stock. This generates several asset pricing features. First, the usual equivalence between marginal q and average q breaks down, as well as the equivalence of investment returns and equity returns. Second, asset prices lead macroeconomic quantities since the latter only reflects part of the state of the economy. Third, asset returns are related to both historical and future book-to-market ratios. Together with future cash flows, this provide a way to directly estimate the lengths of TTB and TTP from firm level data. An empirical examination of COMPUSTAT/CRSP data from March 1984 to December 2009 yields estimates of a 3-quarter TTB ($h = 2$) and a 4-quarter TTP ($d = 3$) in the U.S. economy.

Motivated by risks created and accumulated over these delays, the representative agent uses inventory to facilitate consumption smoothing over time. This imposes a tight connection between inventory and the risk-free rate. In the economy, inventory is a quasi-risk-free asset, and the risk-free rate can be defined as the difference between the marginal rate of substitution between consumption and inventory, and the marginal inventory holding cost. The risk-free rate can be relatively smooth because these two elements are positively correlated. Hence, inventory works as a wedge to disentangle the risk-free rate puzzle from the equity premium puzzle, and helps match volatilities of asset returns. Moreover, TTP is necessary in order to capture inventory holdings observed in the data. Without the TTP constraint, inventory holdings are negligible since the agent can adjust output via investments. One important quantity implication of the delays in production is that these generate short-run cyclicality because the technology frictions are temporary. Consequently, an agent with a preference for the late resolution of uncertainty is necessary to positively price these short-run risks. Additionally, with a small EIS, the procyclical inventory contributes to a counter-cyclical equity premium, as observed in the data. Quantitatively, this model is able to match both macroeconomic quantities and asset prices, the lead-lag patterns between asset prices and macroeconomic quantities, and the return predictability observed in the data. Both TTB and TTP constraints are necessary to match all these dimensions.

This paper highlights the importance of recognizing risk dynamics introduced by technology imperfections and the role of inventory in consumption smoothing. This model can serve as a starting point for several interesting extensions. For example, labor decisions under such technology frictions could be explored. This might help us better understand both labor dynamics and asset pricing (See Uhlig (2009)). Also, an extension to cross-sectional setting would be useful.

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APPENDIX A
THE PERTURBATION PROCEDURES

The perturbation method can be outlined as follows:

- Step 1: Compute the deterministic steady state of these variables.

  From the equilibrium conditions, the deterministic steady state can be solved from the following equations:

$$
\begin{aligned}
0 &= \beta + \frac{1-\upsilon}{\upsilon}\left(\frac{C}{Q}\right)^{\frac{1}{\phi}} - \eta\left(\frac{Q}{K}\right)^{\tau-1} - 1, \\
\sum_{i=0}^{h} \beta^i w_i/\beta^h &= \frac{\beta^{d+1}\left[\alpha K^{\alpha-1}Z^{1-\alpha} - \frac{\eta}{\tau}(1-\tau)\left(\frac{Q}{K}\right)^{\tau}\right]}{1-\beta+\delta\sum_{i=1}^{d+1}\beta^i u_{i-1}}, \\
C &= K^{\alpha}Z^{1-\alpha} - \delta K - \frac{\eta}{\tau}\left(\frac{Q}{K}\right)^{\tau} K, \\
\mu &= (1-\beta)\upsilon\left(\frac{U}{C}\right)^{\frac{1}{\phi}}, \\
q &= \sum_{i=0}^{h}\beta^i w_i/\beta^h, \\
S &= \delta K, \\
I &= \delta K, \\
U &= \left[\upsilon C^{\omega} + (1-\upsilon)Q^{\omega}\right]^{\frac{1}{\omega}}, \\
eu &= U^{1-\gamma}, \\
P &= \frac{\beta}{1-\beta}C, \\
R_f &= 1/\beta,
\end{aligned}
$$

  where all variables without subscripts denote the steady state values, Z is the steady state of the productivity shock and normalized to 1.

- Step 2: Taking the first order derivatives of the equilibrium conditions with respect to the state variables and the perturbation parameter, we apply the steady state values. This gives us 90 equations with 90 distinct derivatives, where 10 derivatives with respect to σ are 0. So, we have 80 unknown derivatives to solve. One challenge here is that this is a highly nonlinear equation system. Since the nonlinear root search algorithm depends on the initial values, to find the non-explosive and economic meaningful solution, I apply several practical tactics in the nonlinear root search:

1. I first solve a standard RBC model with inventory, which gives 27 derivatives as the initial values to feed into the nonlinear root search.
2. For the other 53 derivatives, their initial values are randomly generated. I generate 1000 sets of initial values for these derivatives.
3. I solve the nonlinear equation system for each set of the initial values and record the solutions.
4. I compute and sort distance of all these solutions to the origin. The distance is defined as the sum of the absolute value of each derivative. I also document the number of occurrence of each solution.
5. Finally, starting with the solution of the shortest distance and the most frequent occurrence, I simulate the model to identify the nonexplosive root. The idea here is that solutions closer to the origin are more likely to be stable and the desirable solution occurs repeatedly. Fortunately, the economic meaningful root can usually be obtained after few attempts.

- Step 3: Taking the second order derivatives of the equilibrium conditions with respect to the state variables and the perturbation parameter, we apply the steady state values and the first order derivatives from the last two steps. This gives us 450 equations with 450 distinct derivatives, where 80 derivatives with odd power of σ are 0. So, we have 370 second order derivatives to solve. Note that, due to the chain rule, this is a linear system.
- Step 4: Taking the third order derivatives of the equilibrium conditions with respect to the state variables and the perturbation parameter, we apply the

steady state values and the first- and second-order derivatives from the previous steps. This gives us 1650 equations with 1650 distinct derivatives, where 370 derivatives with odd power of σ are 0. So, we have 1280 third order derivatives to solve. Again, this is a linear system.

I implement the above perturbation procedures in *Mathematica* to take advantage its symbolic computing power and parallel computing capacity. Because the most demanding parts of the computation are the symbolic derivatives and the nonlinear root search, I compute these quantities in parallel. Given the computational demands, the code is executed on a supercomputer at Texas A&M University, the IBM EOS iDataPlex Cluster.

APPENDIX B
EMPIRICAL DATA

I obtain annual market return and the risk-free rate data from the annual Fama-French factors. The real returns are adjusted by the CPI from the NIPA Table 2.3.4. Other macroeconomic data are mainly collected from the NIPA tables. The real output is measured as the real GDP from the NIPA Table 1.1.6. The real consumption is defined as real nondurable goods and services, computed from the NIPA Table 2.3.4 and 2.3.5. The investment is computed as the sum of the real gross private domestic investment (excluding the subcategory of change in private inventories), the government gross investment adjusted by the government gross investment price index, and the personal consumption expenditures on durable goods adjusted by the durable goods price index, computed from the NIPA Table 1.1.6, 3.9.4, 3.9.5, 2.3.4, 2.3.5, 5.6.6A, and 5.6.6B. All these quantities are quarterly and normalized by the civilian noninstitutional population with age over 16, from the Current Population Survey (Serial ID LNU00000000Q). The nominal capital is measured as the fixed assets from the NIPA Table 5.9. The nominal inventory refers to the private inventories (from the NIPA Table 5.7.5A and B). The nominal consumption is the personal consumption expenditures from the NIPA Table 1.1.5. The nominal output is GDP from the NIPA Table 1.1.5. Subject to data availability, these nominal data are annual only. These nominal data are used to compute the ratios of inventory/capital, output/capital, and consumption/capital.

APPENDIX C
NUMERICAL ACCURACY: DIFFERENT ORDER PERTURBATIONS

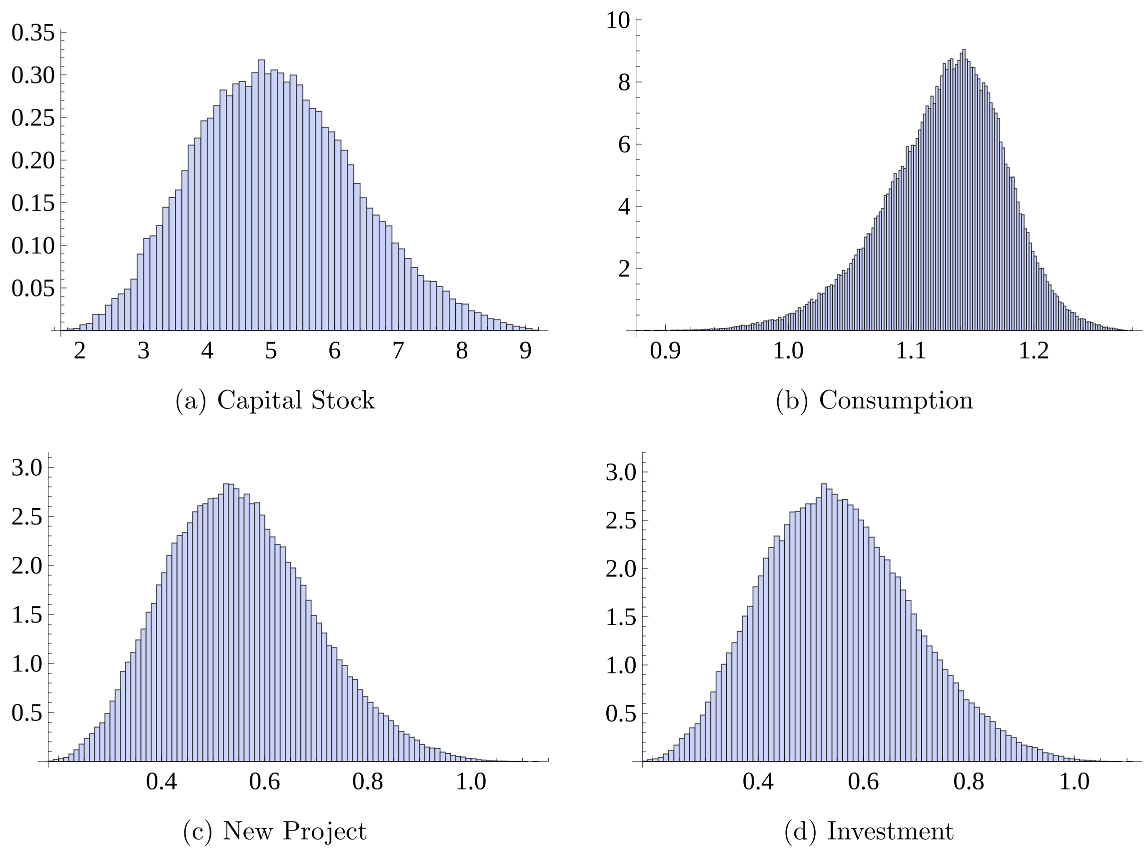
I evaluate the numerical accuracy of different order perturbations in this section.

C.1 Probability Densities

Figure C.1 plots the probability densities of the capital stock, consumption, new projects, investment, and inventory in addition to the pricing kernel, the risk-free rate, and stock returns from a third-order approximation of the main model. All variables are quarterly. The graphs show that all variables are roughly centered around their nonstochastic steady state values. Interestingly, the probability density demonstrates a volatile pricing kernel. This is crucial for asset pricing, since the Hansen-Jagannathan bound requires a volatile pricing kernel for any correctly specified model.

C.2 Policy and Value Functions

I first compare policy functions and value function from a first-, second-, and third-order perturbation in this subsection. Figure C.2 depicts the approximations of consumption, new projects, total investment, inventory, and utility over a capital interval of [50%K, 200%K], which covers almost the entire simulated sample distribution. All other state variables are set to the steady state values, except the capital stock K_t. The results indicate that the first and third order approximations are quite close, while the second order approximation is different. For example, the first and third order approximations show that consumption increases with capital stock; however, the second order approximation shows that consumption decreases with capital stock unless capital stock is small. The same patterns are observed in new projects, inventory, and investment decisions. This point is also clear from



(a) Capital Stock

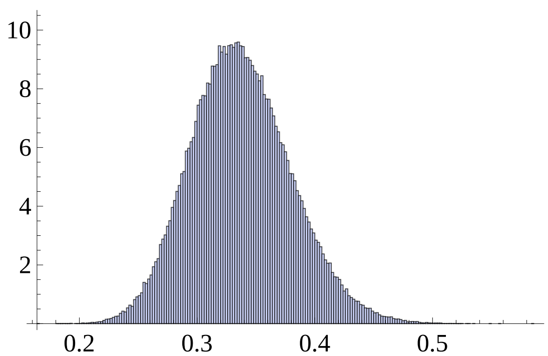
(b) Consumption

(c) New Project

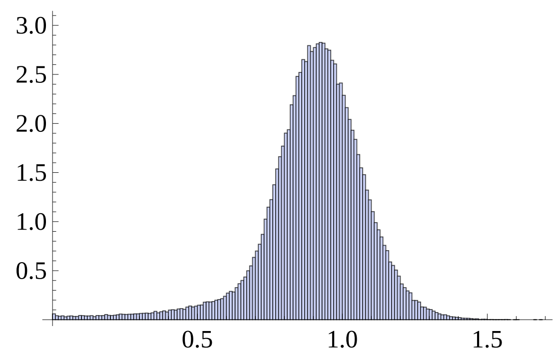
(d) Investment

Fig. C.1.: Probability Densities.

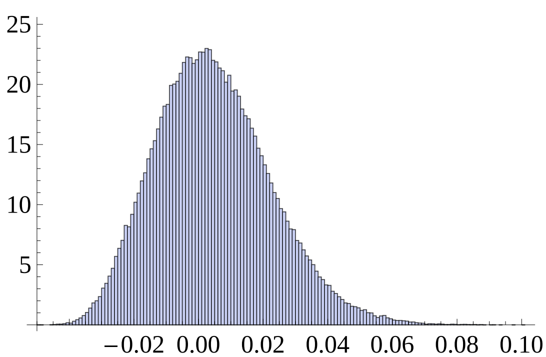
This figure reports the probability densities from a third-order perturbation of the main model. The model is simulated for 1000 paths, and each path has 300 periods while the first 100 periods are discarded as a burn-in. All variables are quarterly.



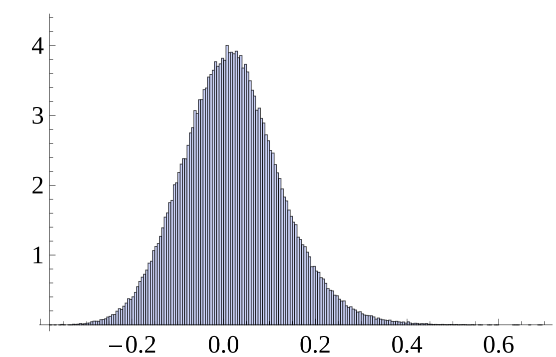
(e) Inventory



(f) Pricing Kernel

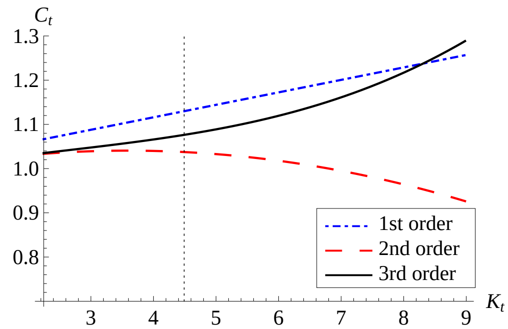


(g) Risk-Free Rate

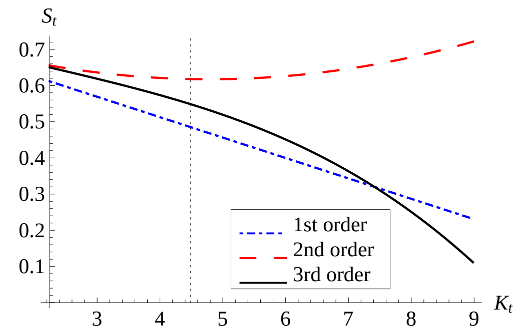


(h) Equity Return

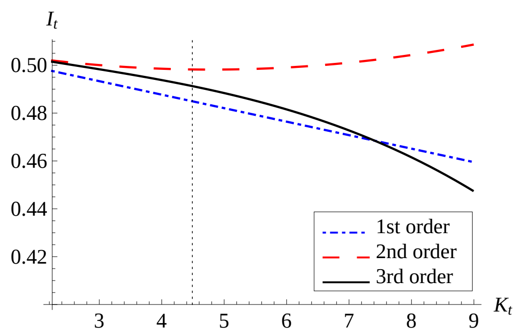
Fig. C.1.: Continued.



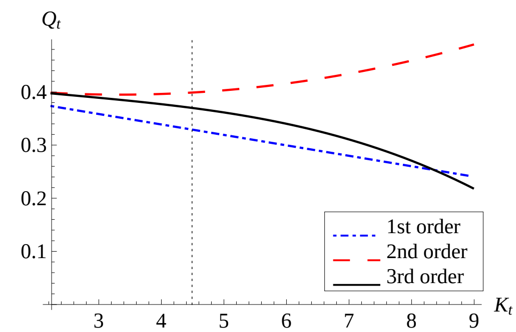
(a) Consumption



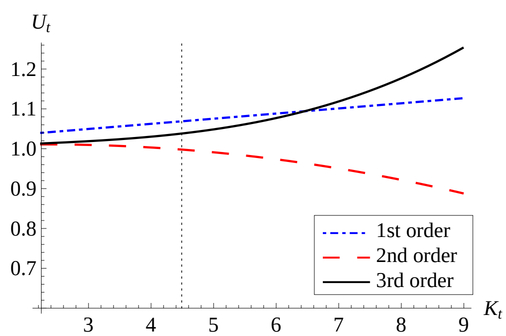
(b) New Project



(c) Investment



(d) Inventory



(e) Utility

Fig. C.2.: Policy Functions and Value Function.

The policy and value functions with respect to the capital stock K_t are plotted, using a first-, second- and third-order perturbation. All other state variables are set to the steady state values.

value function. Utility in Figure C.2 shows that the second order approximation gives counterfactual results, as utility decreases with capital stock. The discrepancy between the second order and the third order approximation widens as capital stock increases.

C.3 Euler Errors

To further evaluate the numerical accuracy of different orders of perturbations, I compute the Euler equation errors, as suggested by Judd and Guu (1997). When $\phi = 1$, the basic asset pricing equation implies that

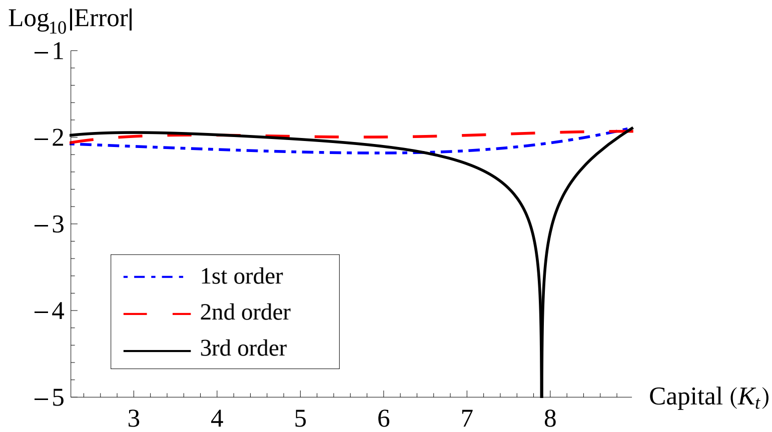
$$\begin{aligned} P_t &= \mathbb{E}_t\left[M_{t,t+1}(P_{t+1} + C_{t+1})\right] \\ &= \mathbb{E}_t\left[\beta \frac{C_t}{C_{t+1}} \left(\frac{C_{t+1}^{\upsilon} Q_{t+1}^{1-\upsilon}}{C_t^{\upsilon} Q_t^{1-\upsilon}}\right)^{\frac{1-\gamma}{\theta}} \left(\frac{U_{t+1}^{1-\gamma}}{eu_t}\right)^{1-\frac{1}{\theta}} (P_{t+1} + C_{t+1})\right]. \end{aligned}$$

Due to approximation errors, the above equation does not hold exactly. The Euler equation error is defined as a fraction of consumption:

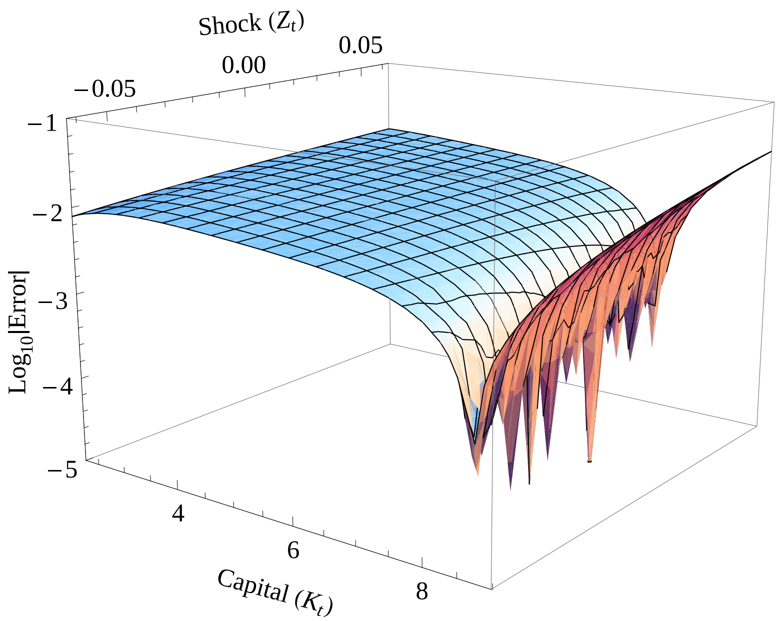
$$\text{Euler Error}_t = 1 - \frac{\left\{\mathbb{E}_t\left[\beta C_{t+1}^{\frac{\upsilon(1-\gamma)}{\theta}-1} \left(\frac{Q_{t+1}}{Q_t}\right)^{\frac{(1-\gamma)(1-\upsilon)}{\theta}} \left(\frac{U_{t+1}^{1-\gamma}}{eu_t}\right)^{1-\frac{1}{\theta}} (P_{t+1} + C_{t+1})\right] \Big/ P_t\right\}^{\frac{1}{\frac{\upsilon(1-\gamma)}{\theta}-1}}}{C_t}.$$

The absolute Euler equation errors are reported in base 10 logarithm in Figure C.3. Therefore, the errors can be interpreted as the percentage of consumption. For example, a value of -2 indicates the error is measured as 1% of consumption. Figure C.3(a) compares the Euler equation errors of different order perturbations for capital K_t over [50%K, 200%K], assuming all other variables are in the steady state and the shock equals 0. Clearly, all of them show almost same Euler equation errors when capital is not too large, but a third-order perturbation performs best when capital is high. Figure C.3(b) displays the Euler equation errors for a third-order perturbation for capital K_t over [50%K, 200%K] and the shock Z_t over $[-3\sigma, 3\sigma]$, which covers the simulated distribution, assuming all other variables are in the steady state. Overall, the Euler equation error is less than 1% for wide ranges of capital and the shock. It performs best when capital is large. Hence, a third-order perturbation only introduces small approximation errors.

Table C.1 reports the simulated moments of macroeconomic quantities and asset prices of different order perturbations to examine to what extent the simulated moments are sensitive to the perturbation order. The macroeconomic quantities for different order perturbations in Table C.1 are quite similar, and all of them closely match the empirical moments. The main discrepancy occurs in the asset prices. First, although the volatilities of equity returns and risk-free rate are similar, the mean asset returns in the first order perturbation are too large. For example, the mean equity return and risk-free rate are 10.30% and 5.93%, respectively, which are much higher than those in the higher order perturbations. This implies that a higher order approximation is necessary in order to capture asset prices. Second, the pricing kernel in the second order perturbation seems unusually large. This is in line with the findings in Figure C.2 where a second order perturbation obtains somewhat counterfactual approximations. Overall, the evidence in Table C.1 shows that a third order perturbation performs well.



(a) Different Order Perturbations



(b) A Third-Order Perturbation

Fig. C.3.: The Euler Equation Error.

Figure (a) and (b) plot the Euler equation errors. Figure (a) compares the Euler equation errors of different order perturbation for capital K_t over [50%K, 200%K], where K is the steady state value, assuming all other variables are in the steady state and shock=0. Figure (b) displays the Euler equation errors of a third-order perturbation for capital K_t over [50%K, 200%K] and shock Z_t over $[-3\sigma, 3\sigma]$, assuming all other variables are in the steady state.

Table C.1: Different Orders of Perturbations: A Comparison.

This table summarizes key moments of macroeconomic quantities and asset prices from calibrations of the main model, using a 1st-, 2nd- and 3rd-order perturbation. The main model is the standard RBC with capital adjustments, inventory, and a 3-quarter ($h = 2$) time-to-build constraint and a 4-quarter ($d = 3$) time-to-produce constraint. The empirical data are from the NIPA tables and the annual Fama-French factors over 1964-2009. The macroeconomic quantities are reported as quarterly, while the asset prices are annualized. The volatilities of output, consumption, and investment are computed from the Hodrick-Prescott filter. All moments are reported in percentages, except the Sharpe ratio and the pricing kernel.

	U.S. Data (1964-2009)	1st Order Perturbation	2nd Order Perturbation	3rd Order Perturbation
Panel A Macroeconomic quantities				
Volatility of output				
$\sigma(Y)$	1.55	1.55	1.55	1.54
Volatility of consumption				
$\sigma(C)$	0.83	0.71	0.68	0.63
Volatility of investment				
$\sigma(I)$	5.28	4.37	3.71	3.75
Mean and volatility of the inventory/consumption ratio				
Q/C	30.68	29.20	29.90	29.91
$\sigma(Q/C)$	7.92	2.80	3.20	3.16
Mean and volatility of the inventory/capital ratio				
Q/K	7.38	8.14	6.94	7.00
$\sigma(Q/K)$	1.69	1.87	1.28	1.29
Panel B Asset prices				
Mean and volatility of the equity returns				
$E[R]$	7.31	10.30	7.99	7.43
$\sigma(R)$	18.50	22.28	24.13	21.36
Mean and volatility of the risk-free rate				
$E[R_f]$	1.73	5.93	1.80	1.86
$\sigma(R_f)$	2.05	2.40	2.40	2.39
Equity premium				
$E[R - R_f]$	5.58	4.37	6.19	5.57
Sharpe ratio				
$E[R - R_f]/\sigma(R)$	0.30	0.20	0.26	0.26
Pricing kernel M				
$\sigma(M)/E(M)$	N/A	0.37	0.66	0.36

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